Equation of Motion 1

Acceleration is the change in velocity per unit time.

$$a = \frac{v - u}{t}$$

from standard grade

$$v - u = a\,t$$

$$\boxed{v = u + a\,t}$$

KEY:

$$v = \text{final speed}\ \ (ms^{-1})$$

$$u = \text{initial speed}\ \ (ms^{-1})$$

$$a = \text{acceleration}\ \ (ms^{-2})$$

$$t = \text{time}\ \ (s)$$

Example 1

A car accelerates from rest to a velocity of 10 ms^{-1} in a time of 4 seconds.

What is its acceleration?

$$u = 0\ ms^{-1}$$
$$v = 10\ ms^{-1}$$
$$t = 4\ s$$
$$a = ?$$

$$v = u + at$$
$$10 = 0 + (a \times 4)$$
$$10 = 4a$$
$$a = \frac{10}{4}$$
$$a = 2.5\ ms^{-2}$$

Example 2

A cyclist accelerates at $0.5ms^{-2}$ for 20 seconds to reach a final velocity of $12ms^{-1}$.

What is the initial velocity?

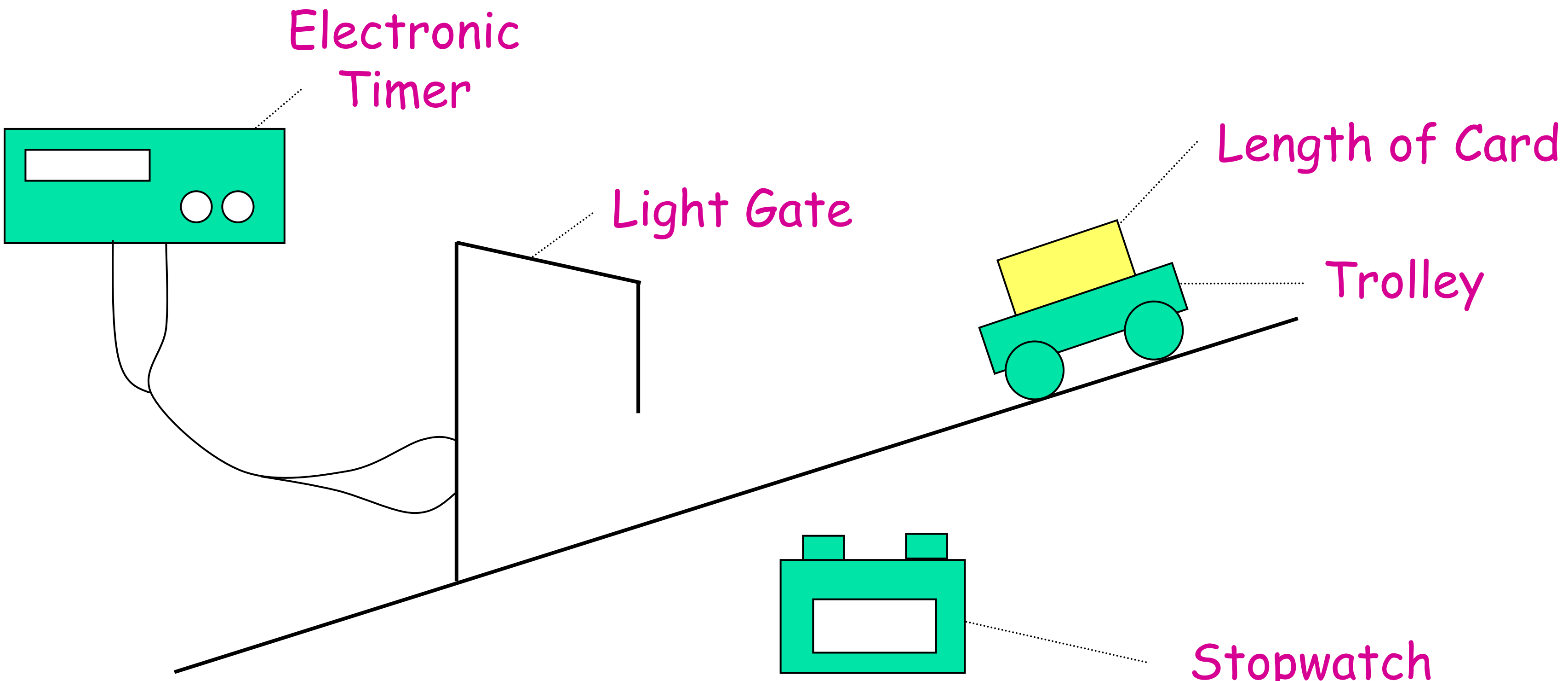
$$a = 0.5\,ms^{-2}$$
$$t = 20\,s$$
$$v = 12\,ms^{-1}$$
$$u = ?$$

$$v = u + at$$
$$12 = u + (0.5 \times 20)$$
$$12 = u + 10$$
$$u = 12 - 10$$
$$\underline{\underline{u = 2\,ms^{-1}}}$$

Worksheet – Equations of Motion 1

Calculating Acceleration 1

Diagram



Measurements

Initial Speed	$0\ ms^{-1}$ (trolley starts from rest).
Time	time taken for trolley and card to reach light gate.
Final Velocity	length of card divided by time taken for card to go through light gate (*instantaneous speed from standard grade*).

Calculation

$$v = u + at$$

$$u = 0\ ms^{-1}$$

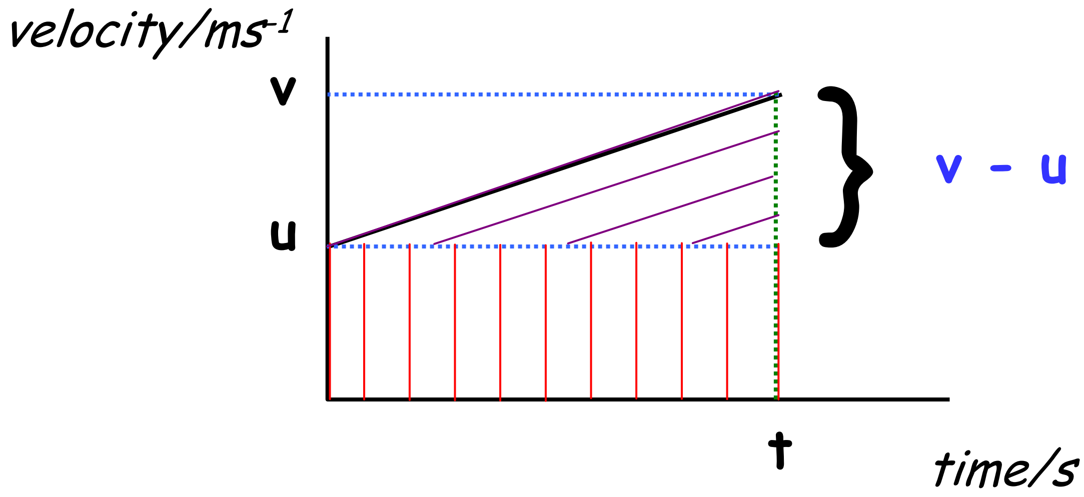
$$t = \_\_\_\_\ s$$

$$v = \frac{\text{length of card}}{\text{time taken}} = \_\_\_\_\_ = \_\_\_\_\_\ ms^{-1}$$

$$a = ?$$

Equation of Motion 2

Consider the following velocity-time graph.



Displacement (s) is the area under the velocity time graph.

$$s = \text{area}\ \square + \text{area}\ \triangle$$

$$= ut + \frac{1}{2}t\left(v - u\right)$$

but from equation (1):

$$v = u + at$$

$$v - u = at$$

so we can rewrite as:

$$s = ut + \frac{1}{2}t\left(at\right)$$

$$s = ut + \frac{1}{2}at^2$$

KEY:

$s =$ displacement (m)

Example 1

An object travelling at $10ms^{-1}$ decelerates at $3ms^{-2}$ for 3s.

Calculate its displacement.

$$u = 10\ ms^{-1}$$
$$a = -3\ ms^{-2}$$
$$t = 3\ s$$
$$s = ?$$

$$s = ut + \frac{1}{2}at^2$$
$$= (10 \times 3) + \left(\frac{1}{2} \times -3 \times 3^2\right)$$
$$= 30 + \left(\frac{1}{2} \times -3 \times 9\right)$$
$$= 30 - 13.5$$
$$s = 16.5\ m$$

Example 2

A car accelerates from rest for 5 seconds, and has a displacement of 50m.

What is its acceleration?

$$u = 0 \text{ ms}^{-1}$$
$$t = 5 \text{ s}$$
$$s = 50 \text{ m}$$
$$a = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$50 = (0 \times 5) + \left(\frac{1}{2} \times a \times 5^2\right)$$

$$50 = 0 + \left(\frac{1}{2} \times a \times 25\right)$$

$$50 = 12.5\,a$$

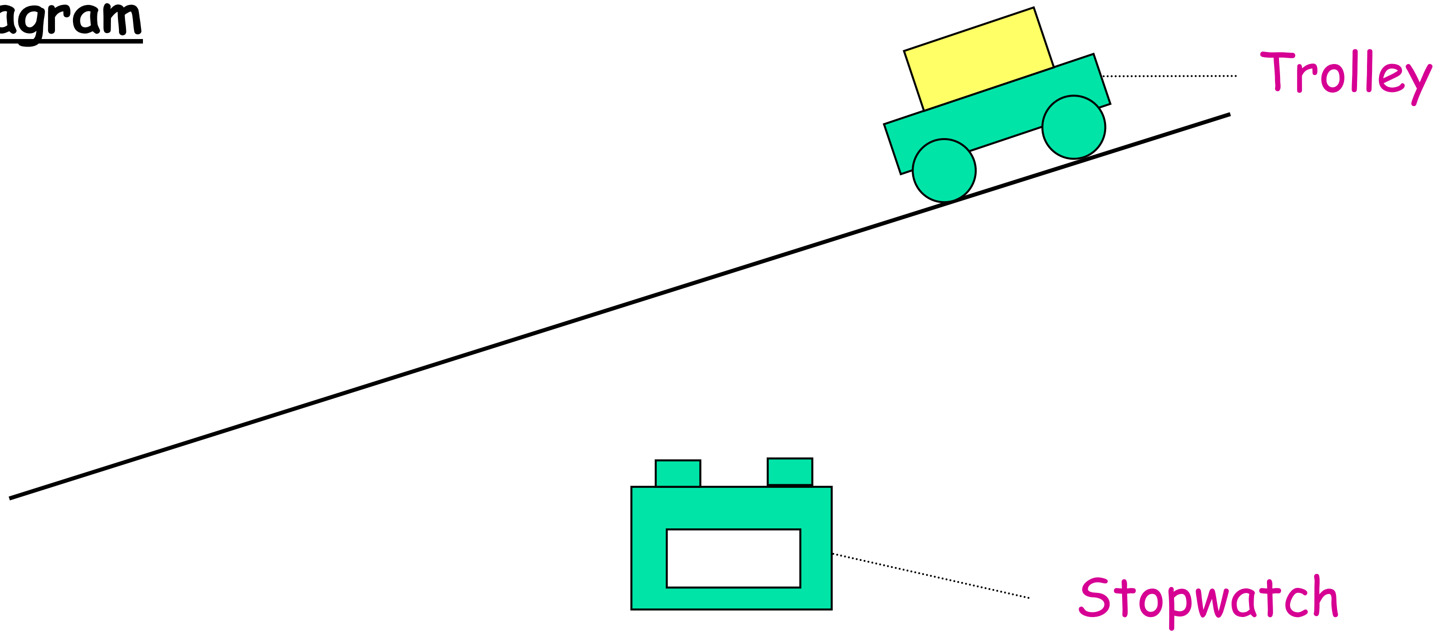
$$a = \frac{50}{12.5}$$

$$\underline{\underline{a = 4 \text{ ms}^{-2}}}$$

Worksheet – Equations of Motion 2

Calculating Acceleration 2

Diagram



Measurements

Displacement	measured using a metre stick.
Time	measures time taken for trolley to travel the measured distance.
Initial Velocity	0 ms^{-1} (trolley starts from rest).

Calculation

$s = 1.2\ m$

$u = 0\ ms^{-1}$

$t = \_\_\_\_\ s$

$a = ?$

$$s = ut + \frac{1}{2}at^2$$

Equation of Motion 3

An equation linking final velocity (v) and displacement (s).

$$v = u + at$$

$$v^2 = (u + at)^2$$

$$v^2 = (u + at)(u + at)$$

$$v^2 = u^2 + 2uat + a^2t^2$$

taking a common factor of 2a gives

$$v^2 = u^2 + 2a\left(ut + \frac{1}{2}at^2\right)$$

and since $s = ut + \frac{1}{2}at^2$

$$v^2 = u^2 + 2as$$

$$s = \text{average velocity} \times \text{time}$$

$$v = u + at$$

$$s = \left(\frac{v+u}{2}\right) t$$

$$t = \frac{v-u}{a}$$

$$s = \left(\frac{v+u}{2}\right) t$$

$$s = \left(\frac{v+u}{2}\right) \left(\frac{v-u}{a}\right)$$

$$s = \frac{v^2 - uv + uv - u^2}{2a}$$

$$s = \frac{v^2 - u^2}{2a}$$

$$v^2 - u^2 = 2as$$

$$v^2 = u^2 + 2as$$

Example 1

A car accelerates at 2 ms^{-2} from an initial velocity of 4 ms^{-1}, and has a displacement of 12m.

Calculate the velocity of the car.

$$a = 2\ ms^{-2}$$
$$u = 4\ ms^{-1}$$
$$s = 12\ m$$
$$v = ?$$

$$v^2 = u^2 + 2as$$
$$= 4^2 + (2 \times 2 \times 12)$$
$$= 16 + 48$$
$$v^2 = 64$$
$$v = \sqrt{64}$$
$$v = 8ms^{-1}$$

Example 2

A motorbike accelerates from rest at $3ms^{-2}$ reaching a final velocity of $12ms^{-1}$.

Calculate the displacement of the motorbike.

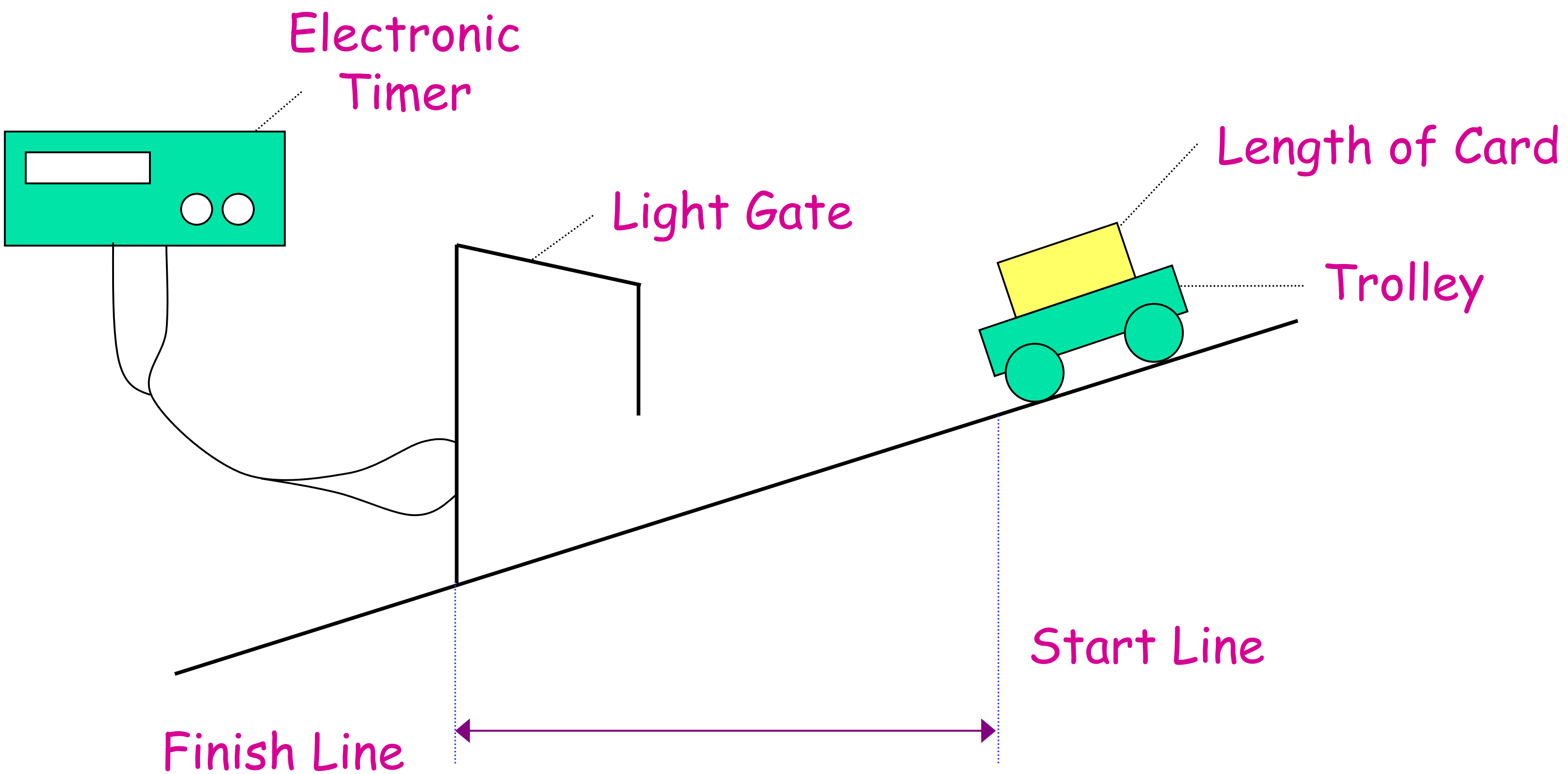
$$u = 0\ ms^{-1}$$
$$a = 3\ ms^{-2}$$
$$v = 12\ ms^{-1}$$
$$s = ?$$

$$v^2 = u^2 + 2as$$
$$12^2 = 0^2 + (2 \times 3 \times s)$$
$$144 = 6s$$
$$\underline{\underline{s = 24\ m}}$$

Worksheet - Equations of Motion 3

Calculating Acceleration 3

Diagram



Measurements

Initial Velocity	0 ms^{-1} (trolley starts from rest).
Displacement	distance from start to finish line.
Final Velocity	length of card divided by time taken for card to go through light gate (*instantaneous speed from standard grade*).

Calculation

$$v^2 = u^2 + 2as$$

$$u = 0 \text{ ms}^{-1}$$

$$s = 0.85 \text{ m}$$

$$v = \frac{\text{length of card}}{\text{time taken}} = \frac{\_\_\_\_\_}{\_\_\_\_\_} = \_\_\_\_\_ \text{ ms}^{-1}$$

$$a = ?$$

Equations of Motion

Example 1

A car accelerates from rest at 3.5 ms^{-2} for 5 seconds.

Calculate (a) its speed.

(b) its distance travelled after 5 seconds.

(a)

$$u = 0\ ms^{-1}$$
$$a = 3.5\ ms^{-2}$$
$$t = 5\ s$$
$$v = ?$$

$$v = u + at$$
$$= 0 + (3.5 \times 5)$$
$$v = 17.5\ ms^{-1}$$

(b)

$s = ?$

$t = 5\ s$

$u = 0\ ms^{-1}$

$a = 3.5\ ms^{-2}$

$$s = ut + \frac{1}{2}at^2$$

$$= (0 \times 5) + \left(\frac{1}{2} \times 3.5 \times 5^2\right)$$

$$s = 43.75m$$

Example 2

A car decelerates from 12ms^{-1} to rest in a time of 5 seconds.

(a) Calculate the deceleration.

(b) Calculate the total braking distance.

(a)

$$u = 12 ms^{-1}$$
$$v = 0\ ms^{-1}$$
$$t = 5\ s$$
$$a = ?$$

$$v = u + at$$
$$0 = 12 + (a \times 5)$$
$$5a = -12$$
$$a = \frac{-12}{5}$$
$$a = -2.4\ ms^{-2}$$

(b)

$s = ?$

$u = 12\ ms^{-1}$

$v = 0\ ms^{-1}$

$a = -2.4\ ms^{-2}$

$$v^2 = u^2 + 2as$$

$$0^2 = 12^2 + (2 \times -2.4 \times s)$$

$$0 = 144 - 4.8s$$

$$4.8s = 144$$

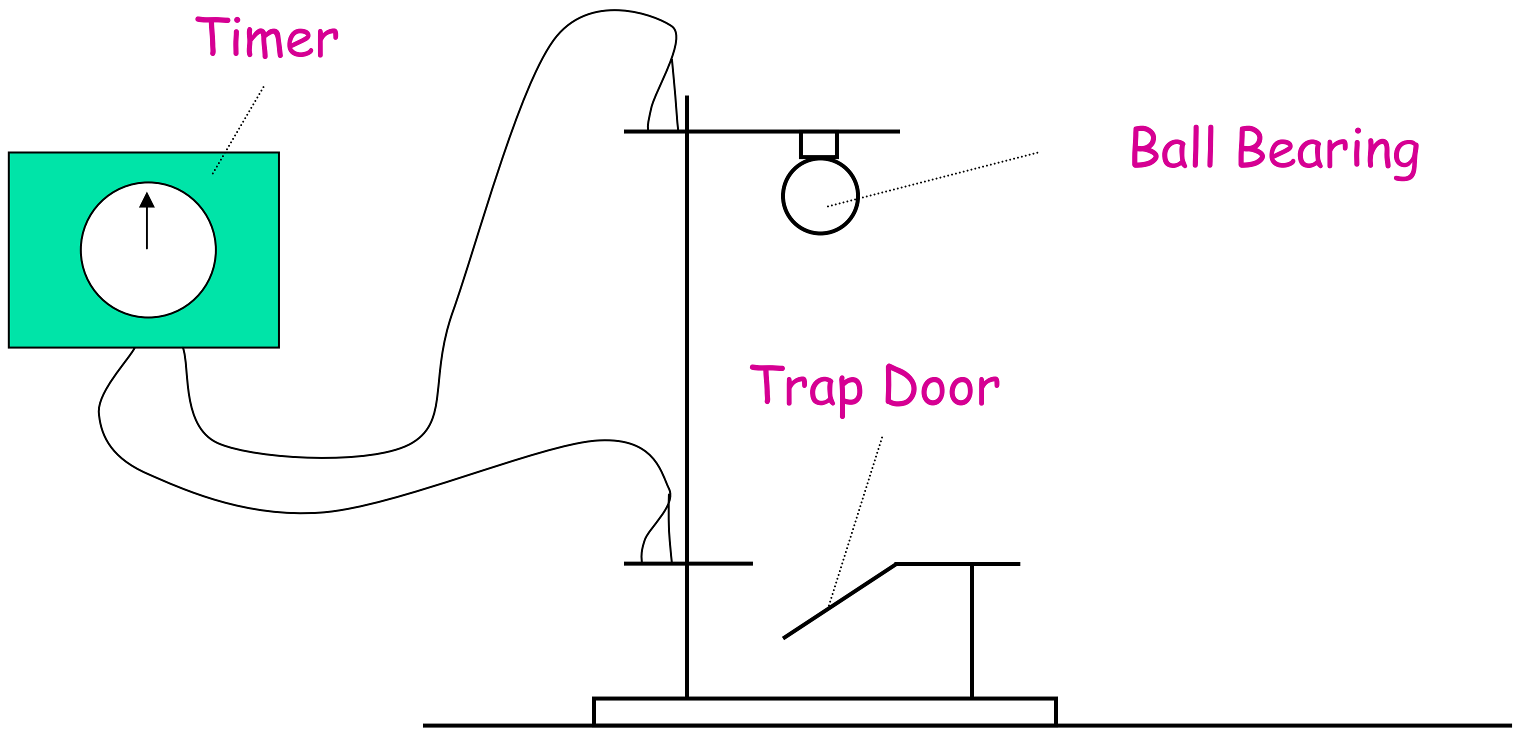
$$s = \frac{144}{4.8}$$

$$s = 30\ m$$

Worksheet – Equations of Motion 4

Acceleration Due To Gravity

Diagram



Measurements

Initial Velocity	0 ms^{-1} (ball bearing starts from rest).
Displacement	distance from the ball bearing to the trap door.
Time	time taken for the ball bearing to drop and reach the trap door.
	timer starts when ball bearing is released, stops when trap door is activated.

Calculation

$$s = ut + \frac{1}{2}at^2$$

$u = 0\ ms^{-1}$

$s =$ ______ m

$t =$ _______ s

$a = ?$

Worksheet - Gravity Problems

Scalars and Vectors

Scalars

A scalar quantity requires only size (magnitude) to completely describe it.

Vectors

A vector quantity requires size (magnitude) and a direction to completely describe it.

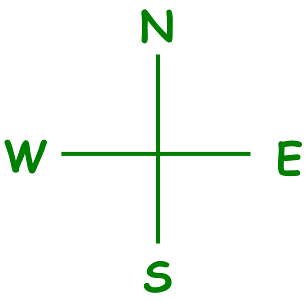
Here are some vector and scalar quantities:

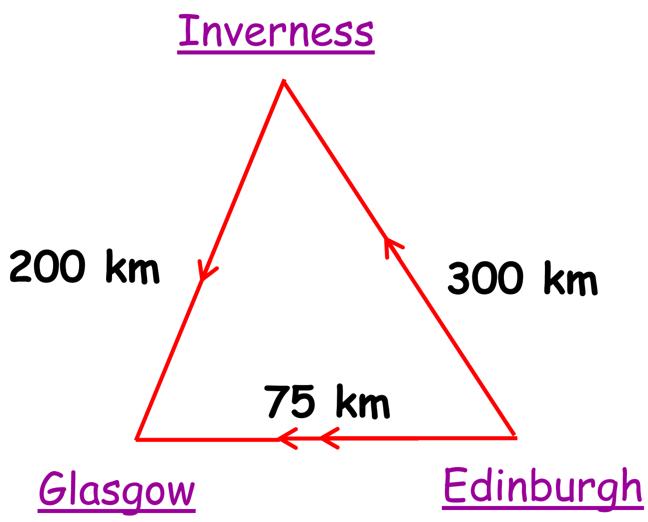
Scalar	**Vector**
time	force
temperature	weight
volume	acceleration
distance	displacement
speed	velocity
energy	
mass	
frequency	
power	

** Familiarise yourself with these scalar and vector quantities **

Distance and Displacement

A helicopter takes off from Edinburgh and drops a package over Inverness before landing at Glasgow as shown.





To calculate how much fuel is needed for the journey, the total distance is required.

distance = 500 km

If the pilot wanted to know his final position relative to his first position, this would be the displacement of the helicopter.

displacement = 75 km due west (270°)

Distance has only size, whereas displacement has both size and direction.

Speed and Velocity

Speed is the rate of change of distance:

$$speed = \frac{distance}{time}$$

Say the helicopter journey lasted 2 hours, the speed would be:

$$speed = \frac{500}{2} = 250\ \mathrm{km\,h^{-1}}$$

Velocity however, is the rate of change of displacement:

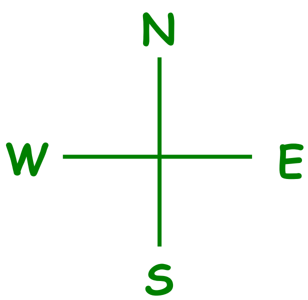
$$\text{velocity} = \frac{\text{displacement}}{\text{time}}$$

So for the 2 hour journey, the velocity is:

$$\text{velocity} = \frac{75}{2} = \underline{\underline{37.5\ \text{km h}^{-1}\ \text{due west } (270^\circ)}}$$

Speed has only size, whereas velocity has both size and direction.

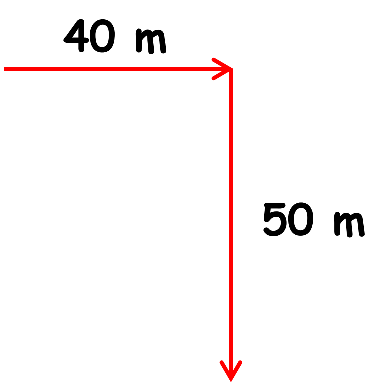
Vector Addition



Example 1

A person walks 40 m east then 50 m south.

(a) draw a diagram showing the journey.

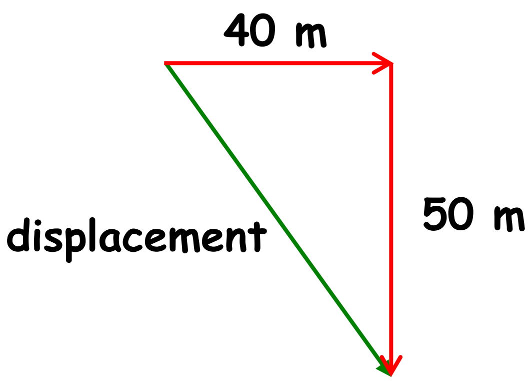


Vectors are joined " tip-to-tail "

(b) Calculate the total distance travelled.

$$\text{distance} = 40 + 50$$
$$= 90\ \text{m}$$

(c) Calculate the total displacement of the person.



The displacement is the size and direction of the line from start to finish.

Size

By Pythagoras:

$$a^2 = b^2 + c^2$$

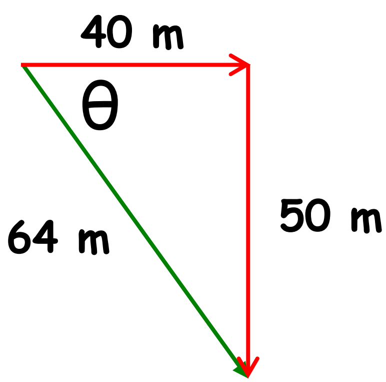
$$(\text{displacement})^2 = 40^2 + 50^2$$

$$= 4100$$

$$\text{displacement} = \sqrt{4100}$$

$$= 64 \text{ m}$$

Direction



$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan\theta = \frac{50}{40}$$

$$\theta = \tan^{-1}(1.25)$$

$$\theta = 51.3^\circ$$

90 + 51.3 = 141.3°
(bearing)

So the total displacement of the person is:

$$s = 64\text{m on a bearing of } 141.3^\circ$$

Example 2

A person walks 40 m east then 50 m south, in a time of 1-minute.

The total distance travelled is 90 m and the displacement is 64 m on a bearing of 141.3° (from example 1).

(a) calculate the speed

$$speed = \frac{distance}{time}$$

$$speed = \frac{90}{60}$$

$$speed = 1.5\ ms^{-1}$$

(b) calculate the velocity of the person

$$\text{velocity} = \frac{\text{displacement}}{\text{time}}$$

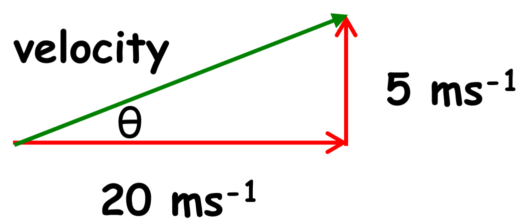
$$\text{velocity} = \frac{64}{60}$$

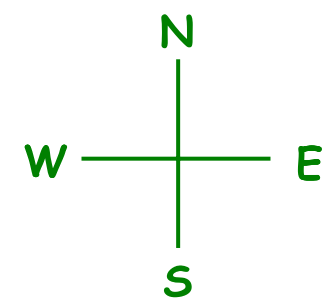
$$\text{velocity} = 1.07\ \text{ms}^{-1} \text{ on a bearing of } 141.3^\circ$$

Example 3

A plane is flying with a velocity of 20 ms^{-1} due east. A crosswind is blowing with a velocity of 5 ms^{-1} due north.

Calculate the resultant velocity of the plane.





Size

By Pythagoras

$$a^2 = b^2 + c^2$$
$$v^2 = 20^2 + 5^2$$
$$= 425$$
$$v = \sqrt{425}$$
$$v = 20.6\ ms^{-1}$$

Direction

$$\tan\theta = \frac{opp}{adj}$$
$$\tan\theta = \frac{5}{20}$$
$$\theta = \tan^{-1}(0.25)$$
$$\theta = 14^\circ$$

90 – 14 = 76°
(bearing)

$$\text{velocity} = 20.6\ ms^{-1} \text{ on bearing of } 76^\circ$$

Q1. A person walks 65 m due south then 85 m due west.

(a) draw a diagram of the journey

(b) calculate the total distance travelled [150 m]

(c) calculate the total displacement. [107 m at bearing of 232.6°]

Q2. A person walks 80 m due north, then 20 m south.

(a) draw a diagram of the journey

(b) calculate the total distance travelled [100 m]

(c) calculate the total displacement. [60 m due north]

Q3. A yacht is sailing at 48 ms^{-1} due south while the wind is blowing at 36 ms^{-1} west.

Calculate the resultant velocity. [60 ms^{-1} on bearing of 216.9°]

Worksheet – Kinematics Problems

Q6

Worksheet – Vector Addition (Using Maths)

Q1 – Q7

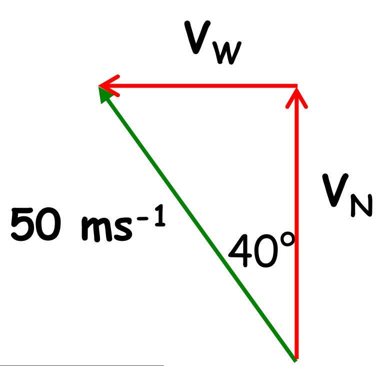
Resolution of Vector into Components

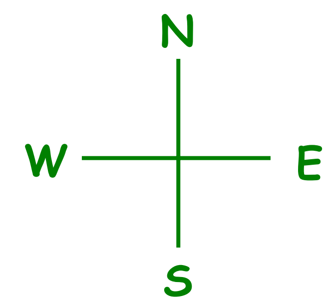
Example 1

A ship is sailing with a velocity of 50 ms^{-1} on a bearing of 320°.

Calculate its component velocity

(a) north





360° - 320° = 40°

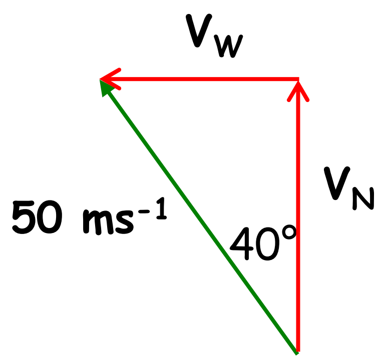
$$\cos\theta = \frac{V_N}{V}$$

$$\cos 40 = \frac{V_N}{50}$$

$$V_N = 50 \times \cos 40$$

$$V_N = 38.3\ ms^{-1}$$

(b) west



$$\sin\theta = \frac{V_W}{V}$$

$$\sin 40 = \frac{V_W}{50}$$

$$V_W = 50 \times \sin 40$$

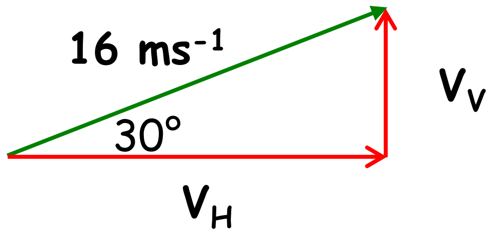
$$V_W = 32.1\,\text{ms}^{-1}$$

Example 2

A ball is kicked with a velocity of 16 ms^{-1} at an angle of 30° above the ground.

Calculate the horizontal and vertical components of the balls velocity.

Horizontal



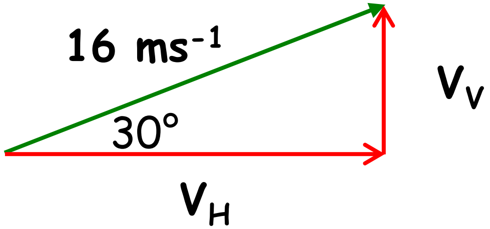
$$\cos\theta = \frac{V_H}{V}$$

$$\cos 30 = \frac{V_H}{16}$$

$$V_H = 16 \times \cos 30$$

$$V_H = 13.9\ ms^{-1}$$

Vertical



$$\sin\theta = \frac{V_V}{V}$$

$$\sin 30 = \frac{V_V}{16}$$

$$V_V = 16 \times \sin 30$$

$$V_V = 8\ ms^{-1}$$

Worksheet – Resolution of Vectors

Q1 - Q5

Worksheet – Kinematics Problems

Q10 + Q15

Direction

So far we have only considered motion in one direction.

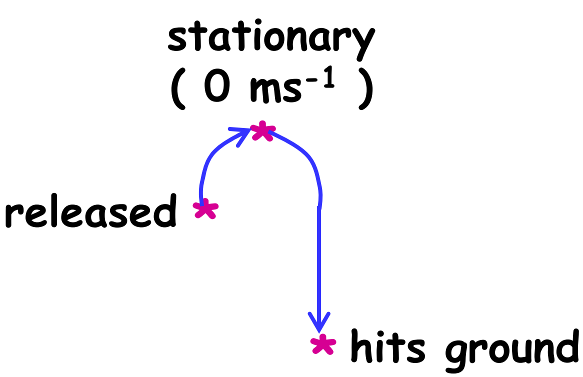
Therefore there has been no noticeable difference between speed and velocity.

Example

A helicopter is travelling upwards with a velocity of 25 ms^{-1}.

A package is released and hits the ground 14 s later.

Path of Package



This example poses a new problem as there is motion in two directions.

It is necessary to distinguish between the two directions.

One motion is positive, the other is negative.

Choose the upward direction as positive!

(a) How long will it take the package to reach its maximum height? (2)

$$t = ?$$
$$u = +25\ ms^{-1}$$
$$v = 0\ ms^{-1}$$
$$a = -9.8\ ms^{-2}$$

$$v = u + at$$
$$0 = 25 + (-9.8 \times t)$$
$$9.8\ t = 25$$
$$t = 2.55\ s$$

(b) How high as it climbed since being released? (2)

$s = ?$

$u = +25\ ms^{-1}$

$t = 2.55\ s$

$a = -9.8\ ms^{-2}$

$$s = ut + \frac{1}{2}at^2$$

$$= (25 \times 2.55) + \frac{1}{2}(-9.8 \times 2.55^2)$$

$$= 63.75 + (-31.86)$$

$$s = 31.9\ m$$

(c) Calculate the velocity of the package just before it hits the ground. (2)

$v = ?$

$u = +25\ ms^{-1}$

$t = 14\ s$

$a = -9.8\ ms^{-2}$

$$v = u + at$$

$$= 25 + (-9.8 \times 14)$$

$$= 25 + (-137.2)$$

$$v = -112.2\ ms^{-1}$$

The negative indicates travelling downwards.

(d) How high above the ground is the helicopter when the package is released? (2)

$s = ?$

$u = +25\ ms^{-1}$

$t = 14\ s$

$a = -9.8\ ms^{-2}$

$$s = ut + \frac{1}{2}at^2$$

$$= (25 \times 14) + \frac{1}{2}(-9.8 \times 14^2)$$

$$= 350 + (-960.4)$$

$$s = -610.4\ m$$

So the helicopter is 610.4 m above the ground.

Worksheet – Kinematics Problems

Problems on the Equations of Motion

Q23, 25, 26, 31, 32, 34, 37, 40 – 50.

Motion Graphs

An object is dropped from rest and falls through the air for 4 seconds.

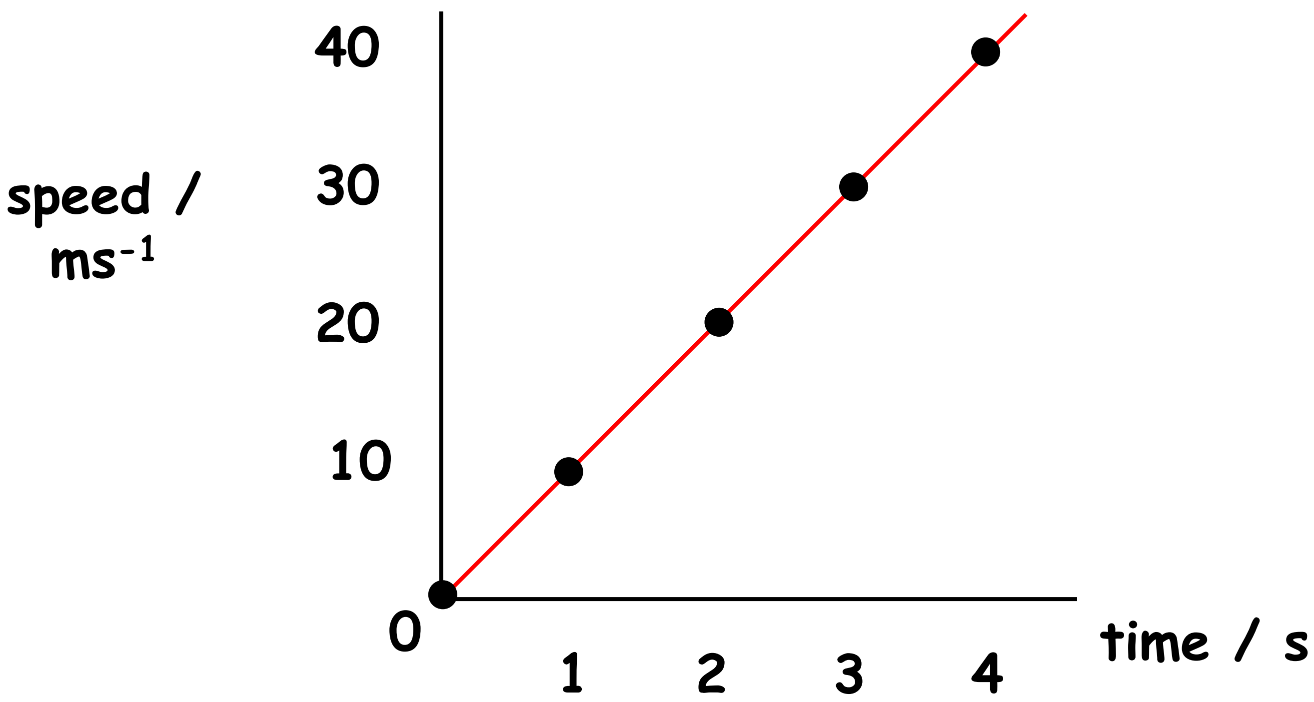
Draw three graphs of the objects subsequent motion.

1. A velocity - time graph.
2. A displacement – time graph.
3. An acceleration – time graph.

Graph 1

Calculate the velocity at times 0s, 1s, 2s, 3s and 4s.

Time (s)	0	1	2	3	4
Velocity (ms^{-1})	0	9.8	19.6	29.4	39.2



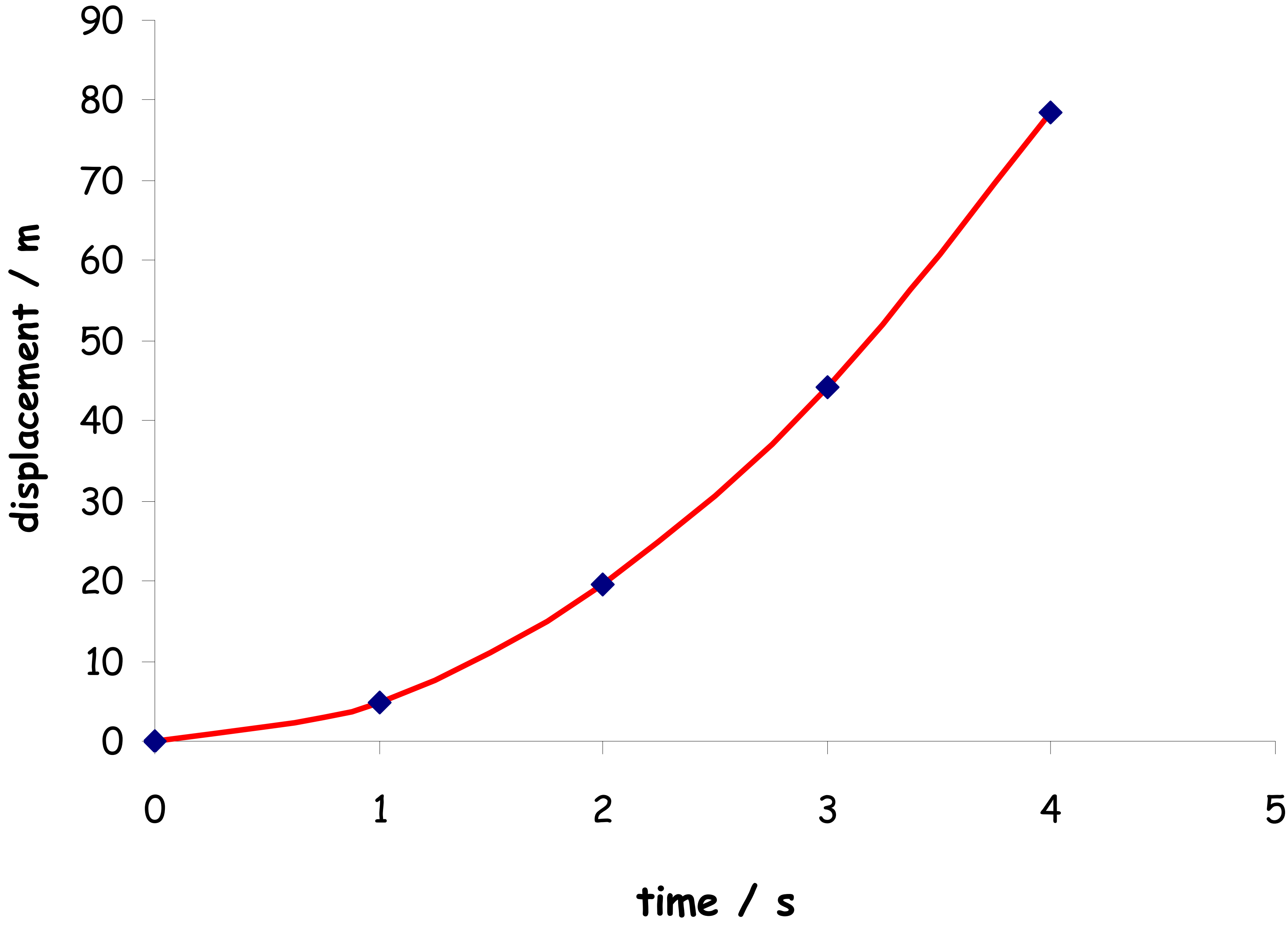
Graph 2

Calculate the displacement (distance travelled) after 0s, 1s, 2s, 3s and 4s.

To calculate displacement, need the equation

$$s = ut + \frac{1}{2}at^2$$

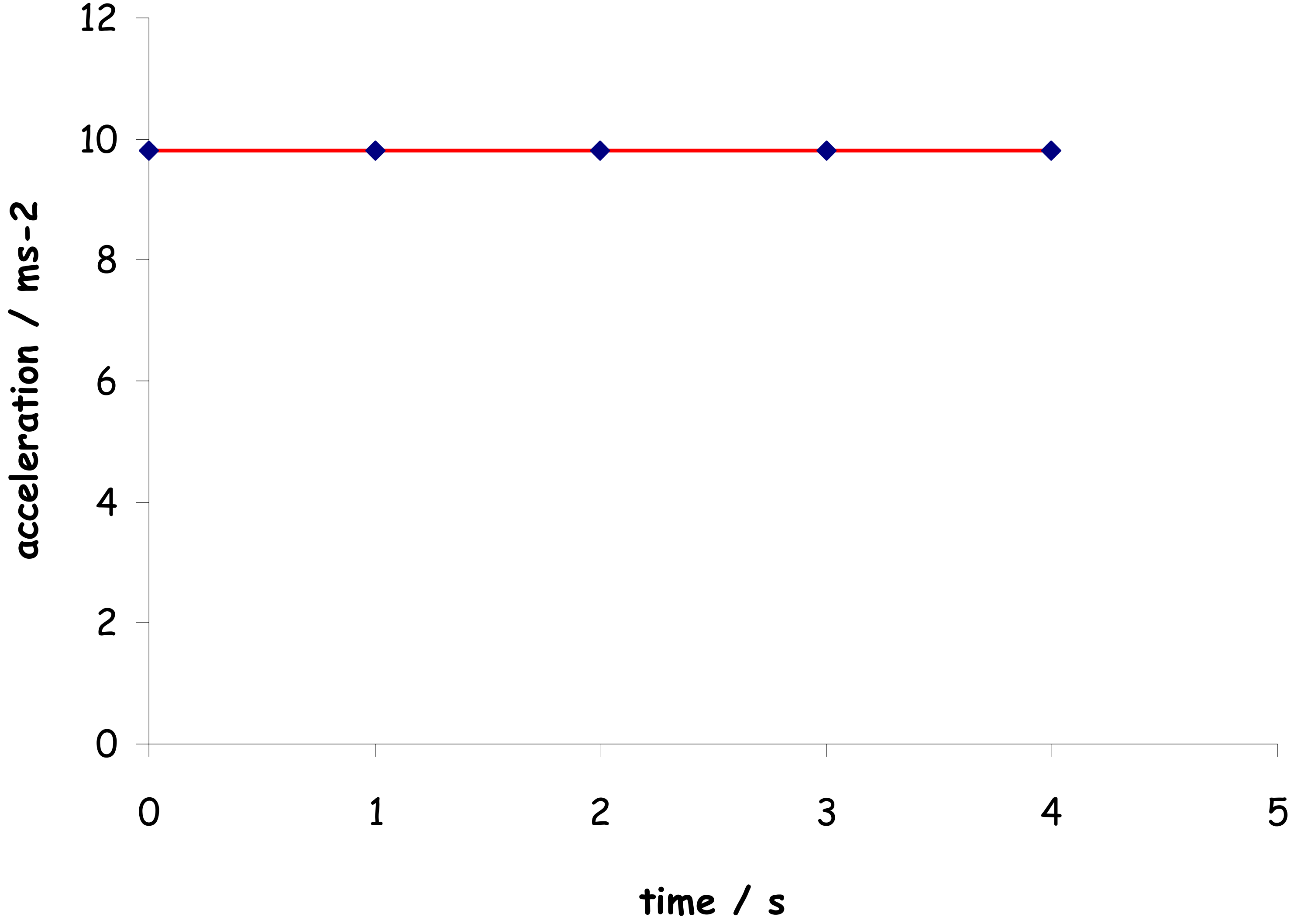
Time (s)	0	1	2	3	4
Displacement (m)	**0**	**4.9**	**19.6**	**44.1**	**78.4**


90
80
70
60
50
40
30
20
10
0
displacement / m
0
1
2
3
4
5
time / s

Graph 3

The only acceleration on the object, is the acceleration due to gravity.

So the rate of acceleration is $9.8\ ms^{-2}$.

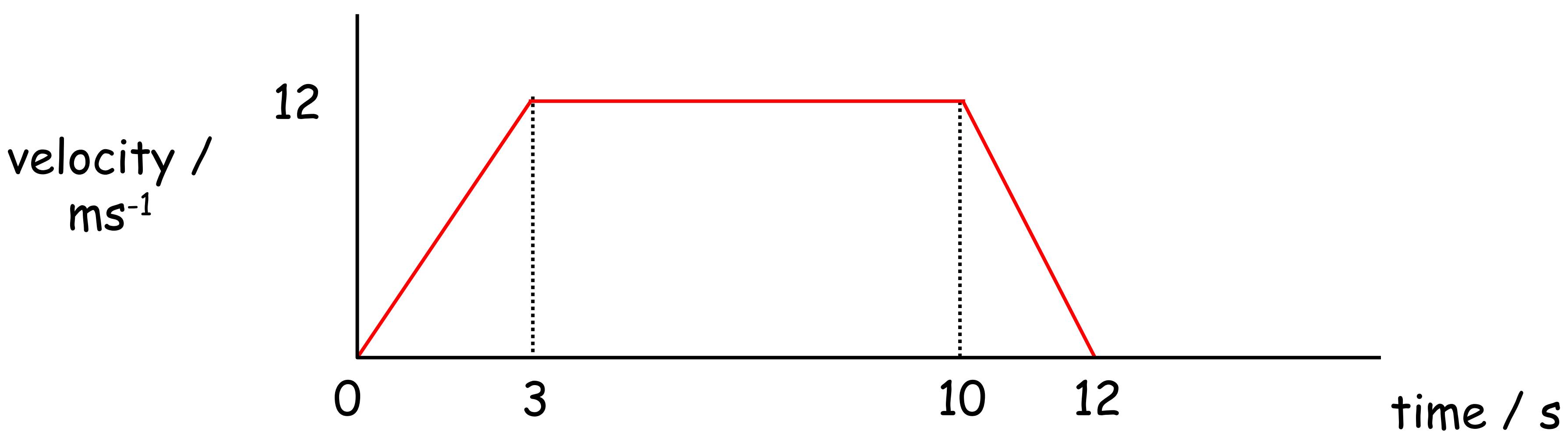


Graphs

Each velocity – time graph has a corresponding acceleration – time graph and displacement- time graph.

Example

The following velocity – time graph describes a journey.



Draw the corresponding acceleration – time graph.

0-3 seconds

$$a = \frac{v-u}{t}$$

$$= \frac{12-0}{3}$$

$$a = 4\,ms^{-2}$$

3-10 seconds

$$a = \frac{v-u}{t}$$

$$= \frac{12-12}{7}$$

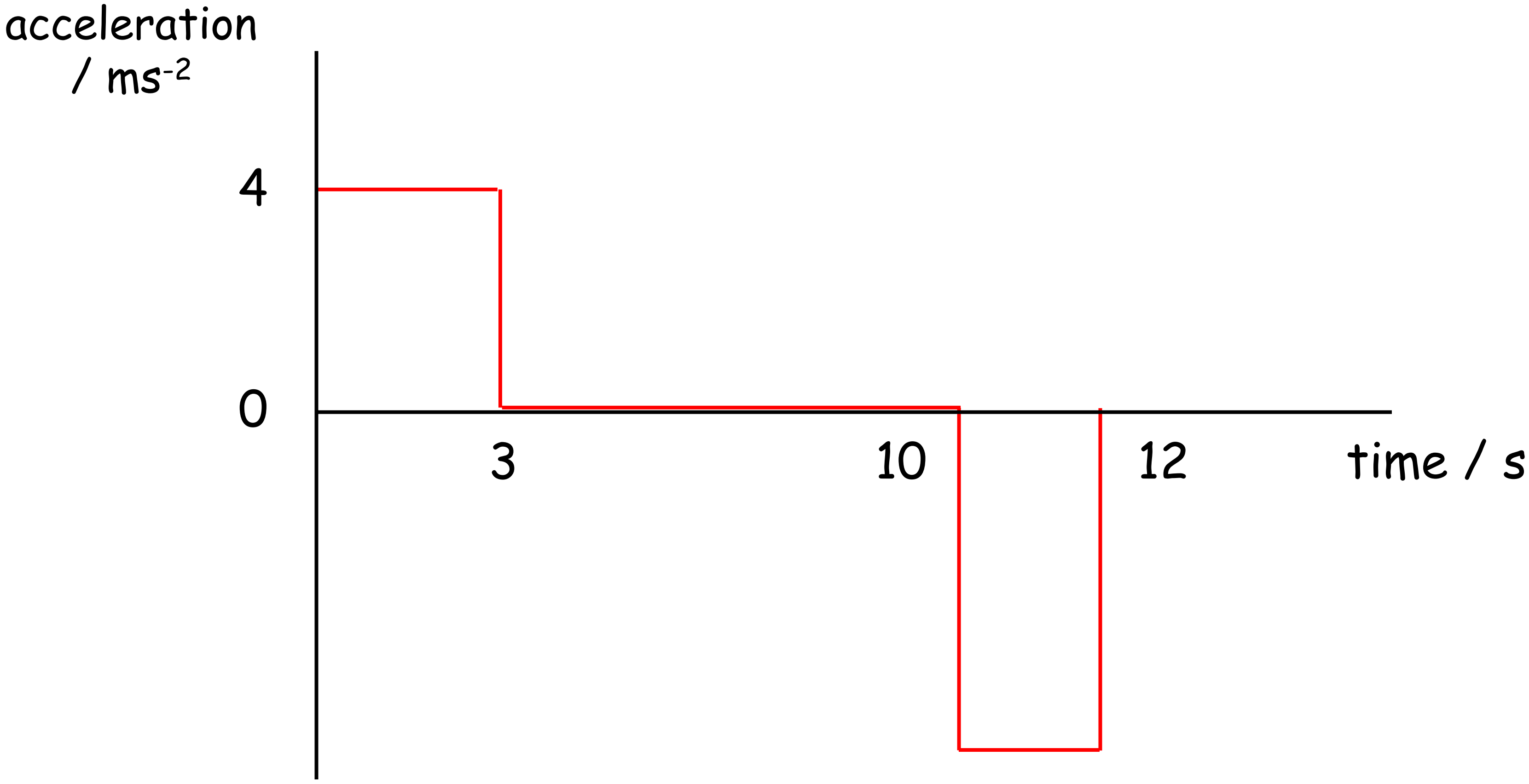
$$a = 0\,ms^{-2}$$

10-12 seconds

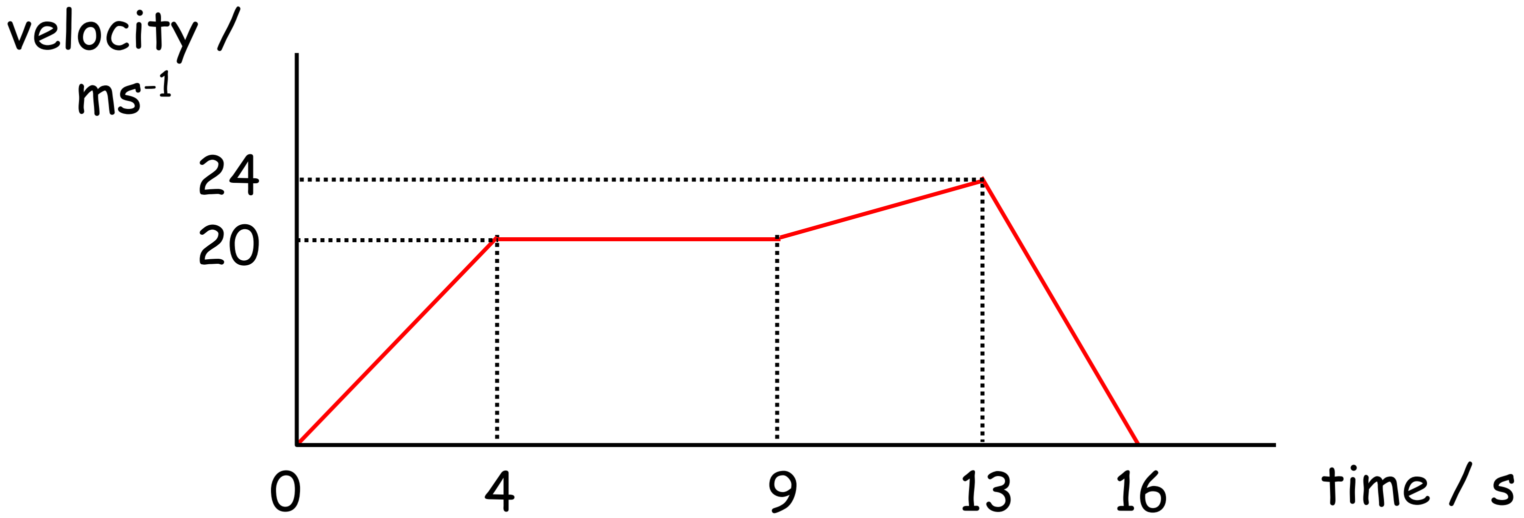
$$a = \frac{v-u}{t}$$

$$= \frac{0-12}{2}$$

$$a = -6\,ms^{-2}$$


acceleration
/ ms-2
4
0
3
10
12
time / s

Q1. Copy out the following velocity - time graph and underneath it draw the corresponding acceleration – time graph.



0-4 seconds

$$a = \frac{v - u}{t}$$

$$= \frac{20 - 0}{4}$$

$$a = 5\ ms^{-2}$$

4-9 seconds

$$a = 0\ ms^{-2}$$

9-13 seconds

$$a = \frac{v - u}{t}$$

$$= \frac{24 - 20}{4}$$

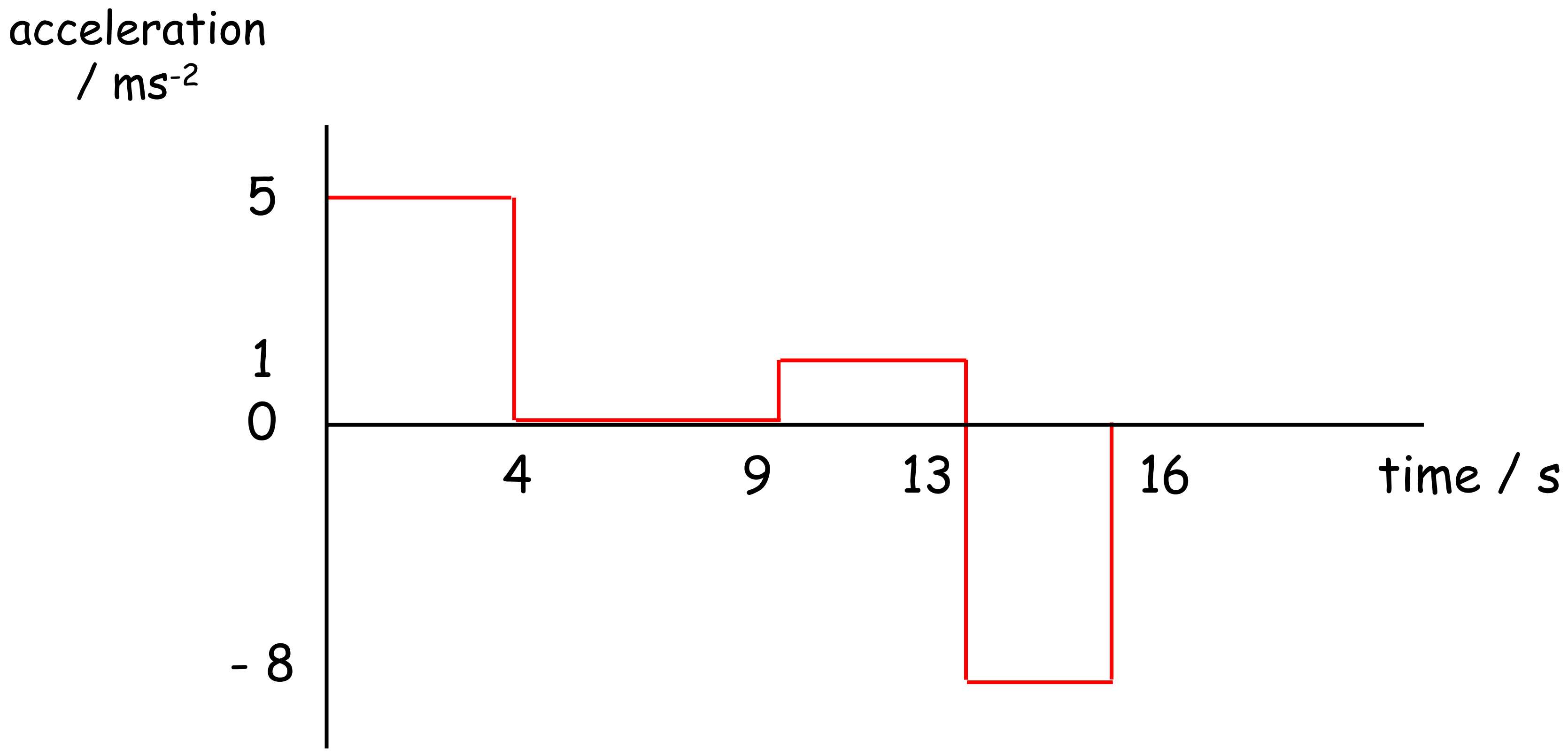
$$a = 1\ ms^{-2}$$

13-16 seconds

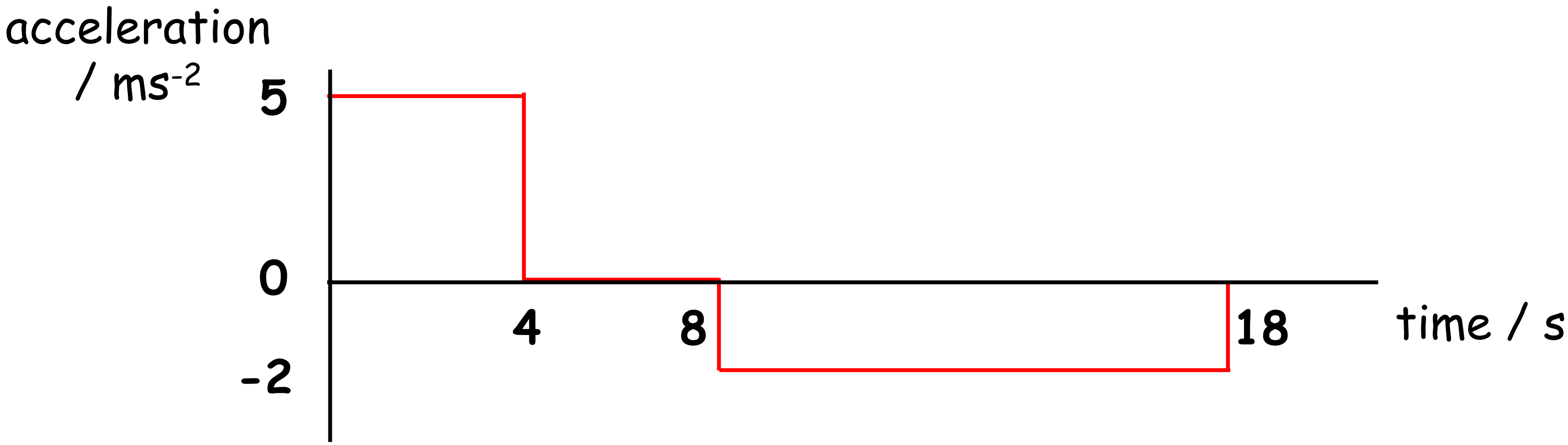
$$a = \frac{v - u}{t}$$

$$= \frac{0 - 24}{3}$$

$$a = -8\ ms^{-2}$$

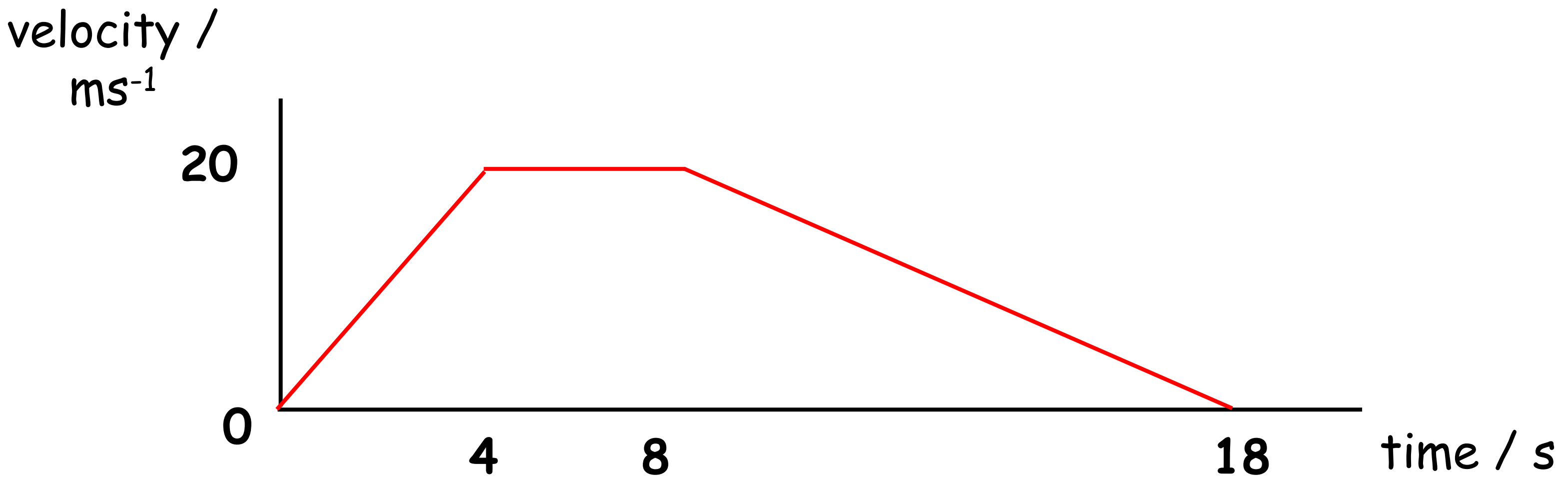

acceleration / ms-2
5
1
0
- 8
4
9
13
16
time / s

Q2. Using the following acceleration – time graph of an object starting from rest, draw the corresponding velocity – time graph.

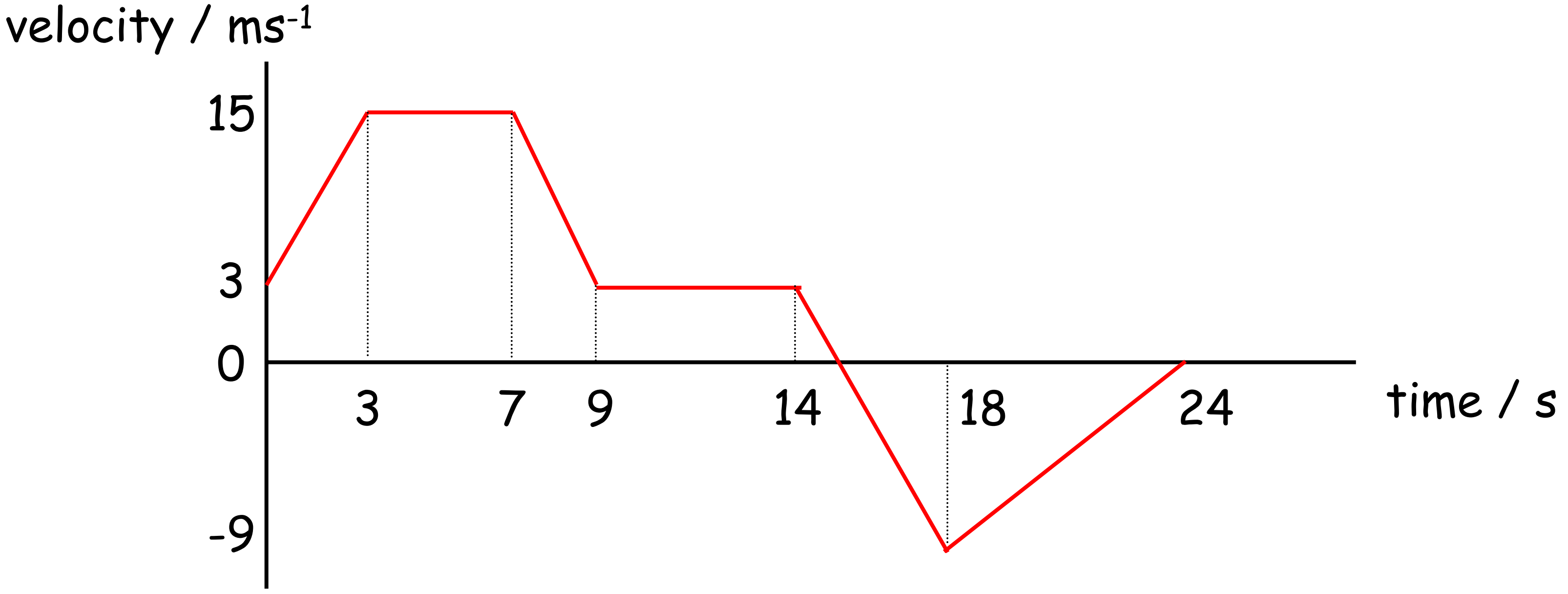


$$v = u + at$$
$$= 0 + (5 \times 4)$$
$$v = 20\ ms^{-1}$$

$$v = u + at$$
$$= 20 + (-2 \times 10)$$
$$= 20 - 20$$
$$v = 0\ ms^{-1}$$


velocity / ms-1
20
0
4
8
18
time / s

Q3. Copy out the following velocity – time graph and underneath it draw the corresponding acceleration – time graph (after appropriate calculations).



0-3 seconds

$$a = \frac{v - u}{t}$$

$$= \frac{15 - 3}{3}$$

$$a = 4\ ms^{-2}$$

3-7 seconds

$$a = 0\ ms^{-2}$$

7-9 seconds

$$a = \frac{v - u}{t}$$

$$= \frac{3 - 15}{2}$$

$$a = -6\ ms^{-2}$$

9-14 seconds

$$a = 0\ ms^{-2}$$

14-18 seconds

$$a = \frac{v - u}{t}$$

$$= \frac{-9 - 3}{4}$$

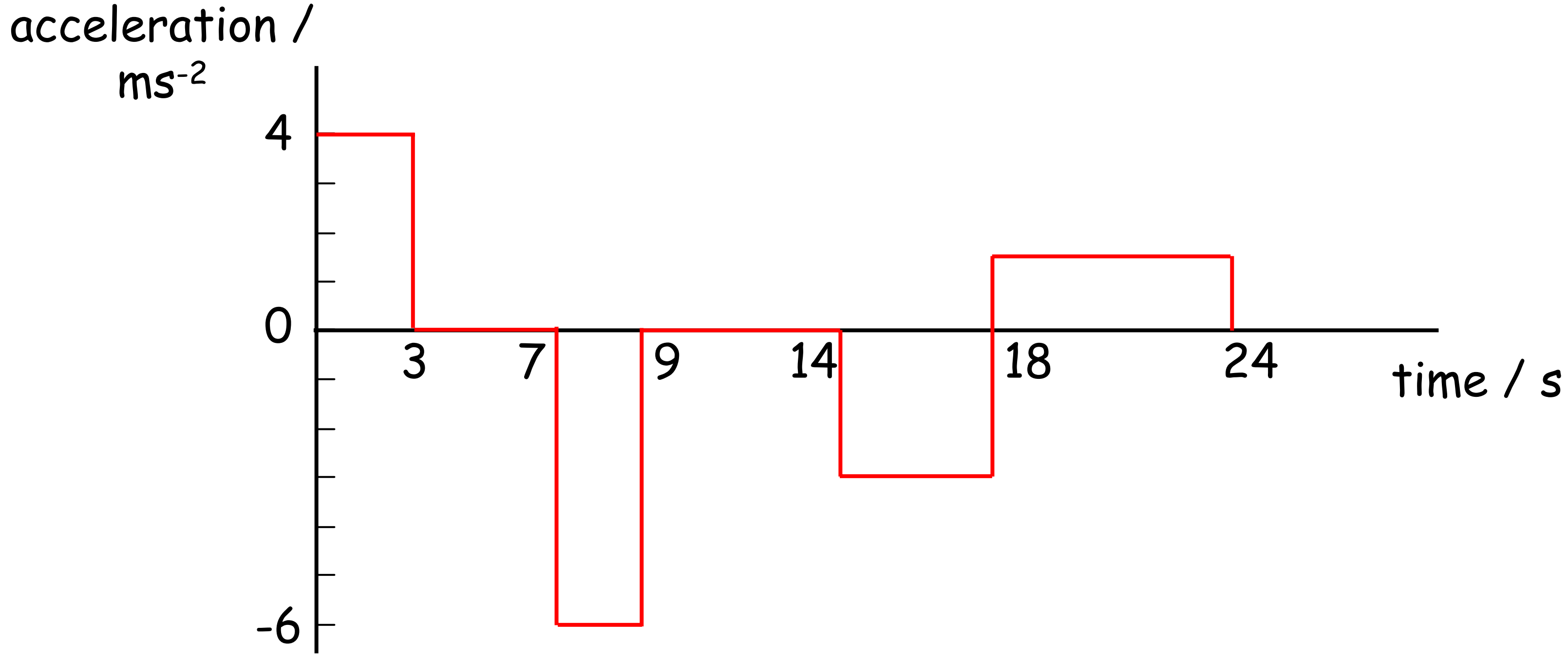
$$a = -3\ ms^{-2}$$

18-24 seconds

$$a = \frac{v - u}{t}$$

$$= \frac{0 - (-9)}{6}$$

$$a = 1.5\ ms^{-2}$$


acceleration / ms^{-2}
4
0
-6
3
7
9
14
18
24
time / s

Worksheet - Kinematics Problems

Graphs

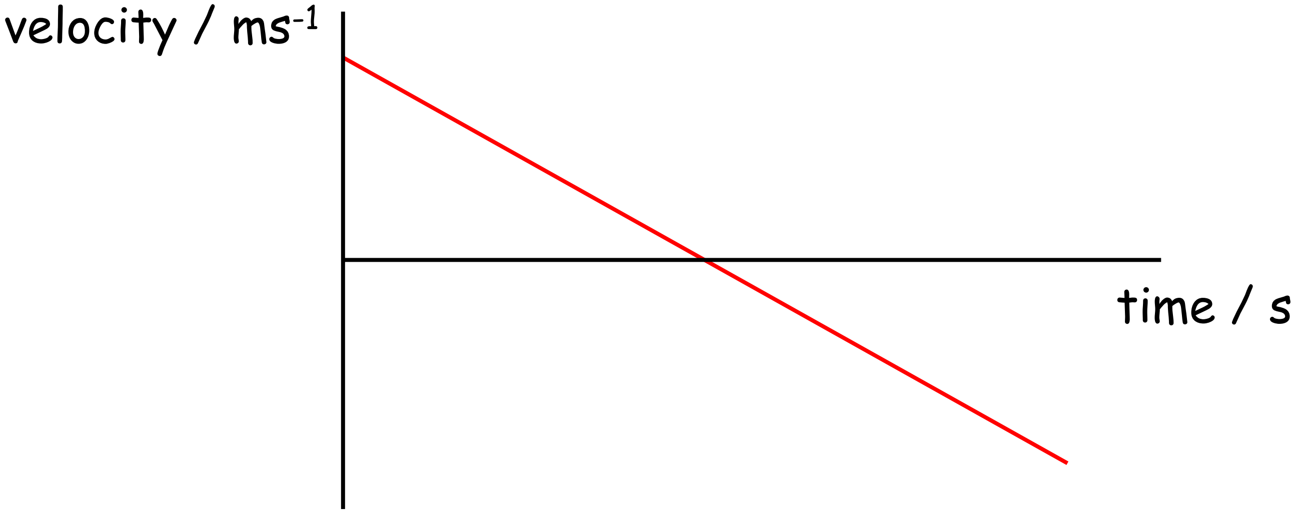
Q3, Q4, Q6, Q7 & Q8

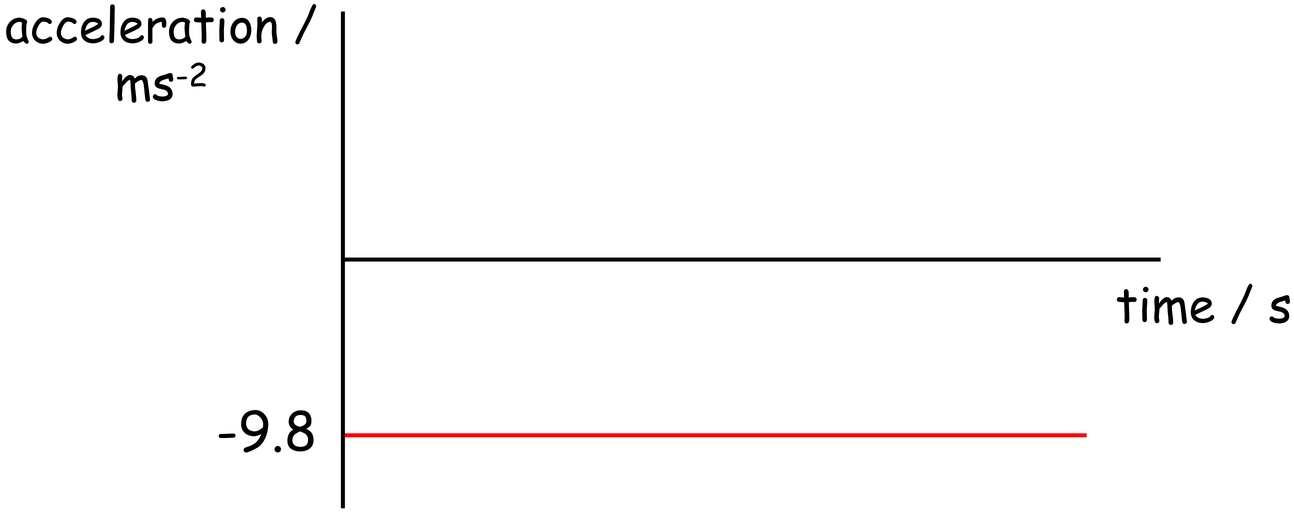
Ball Thrown Into Air

A ball is thrown directly upwards into the air.

It rises into the air and falls back down to the thrower.

The velocity – time graph and corresponding acceleration – time graph are shown.



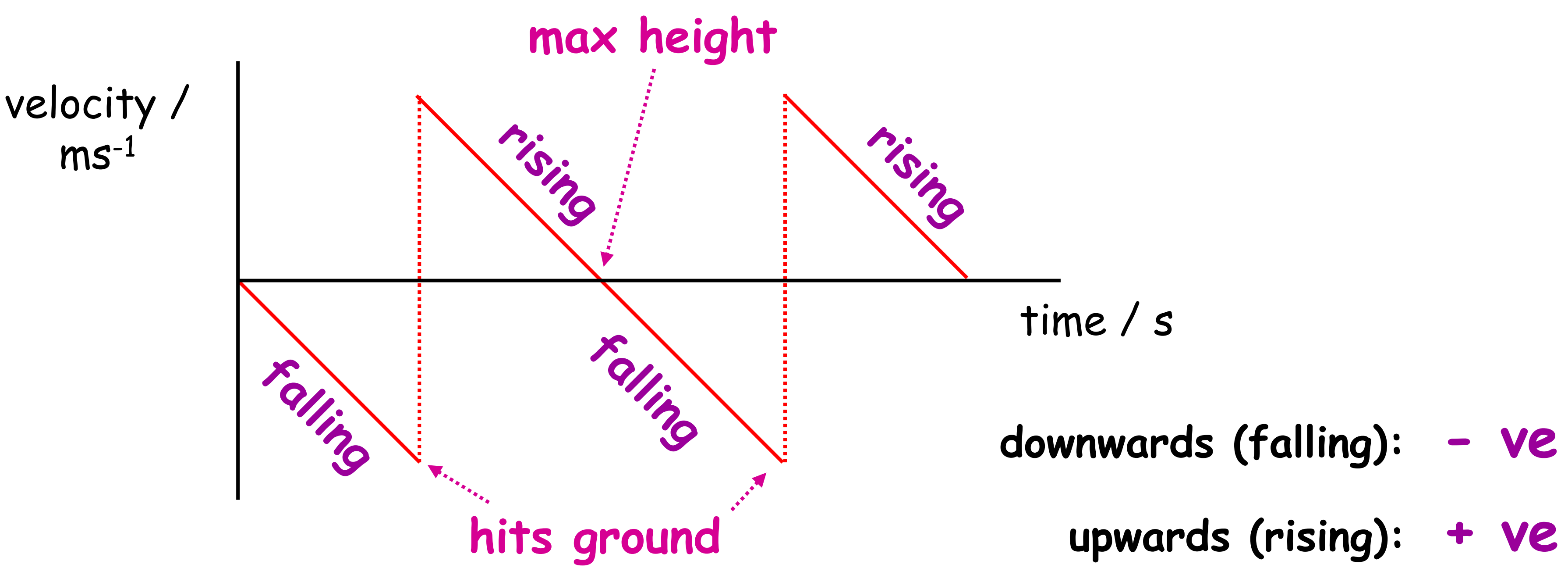

acceleration / ms-2
time / s
-9.8

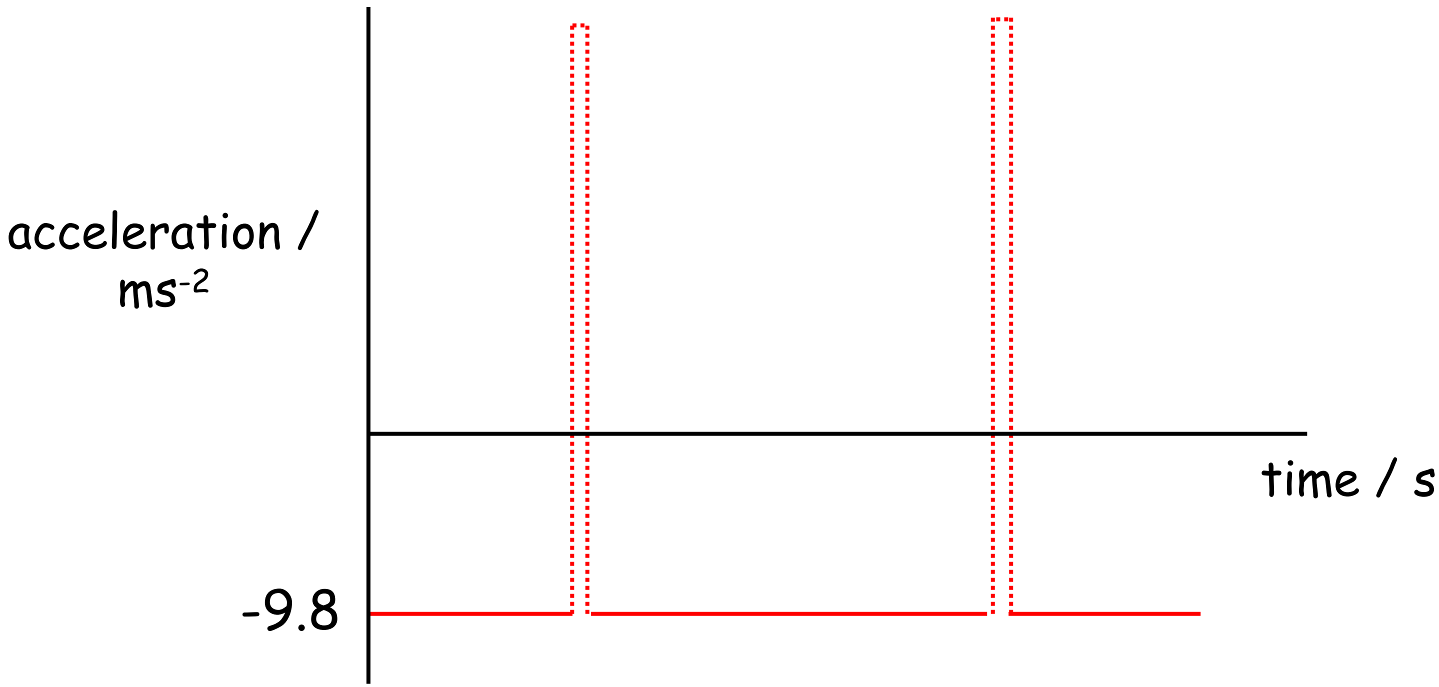
Bouncing Ball (No Energy Loss)

A ball is dropped from a height to the ground.

The ball bounces twice with no energy loss and is then caught.

The velocity - time and acceleration - time graphs are as follows:



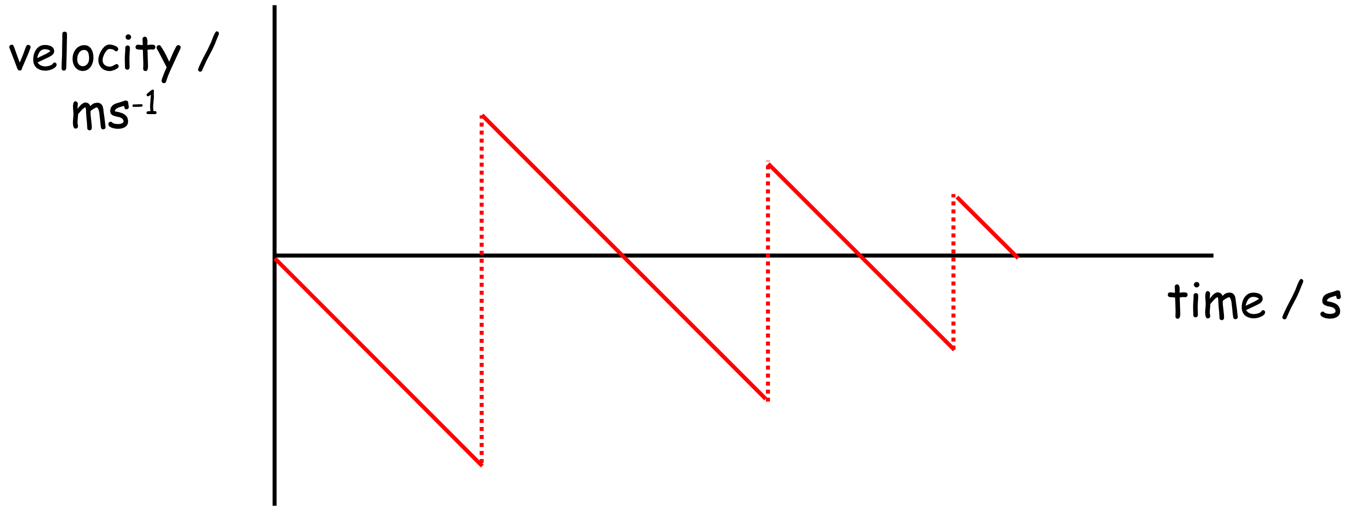

acceleration / ms-2
-9.8
time / s

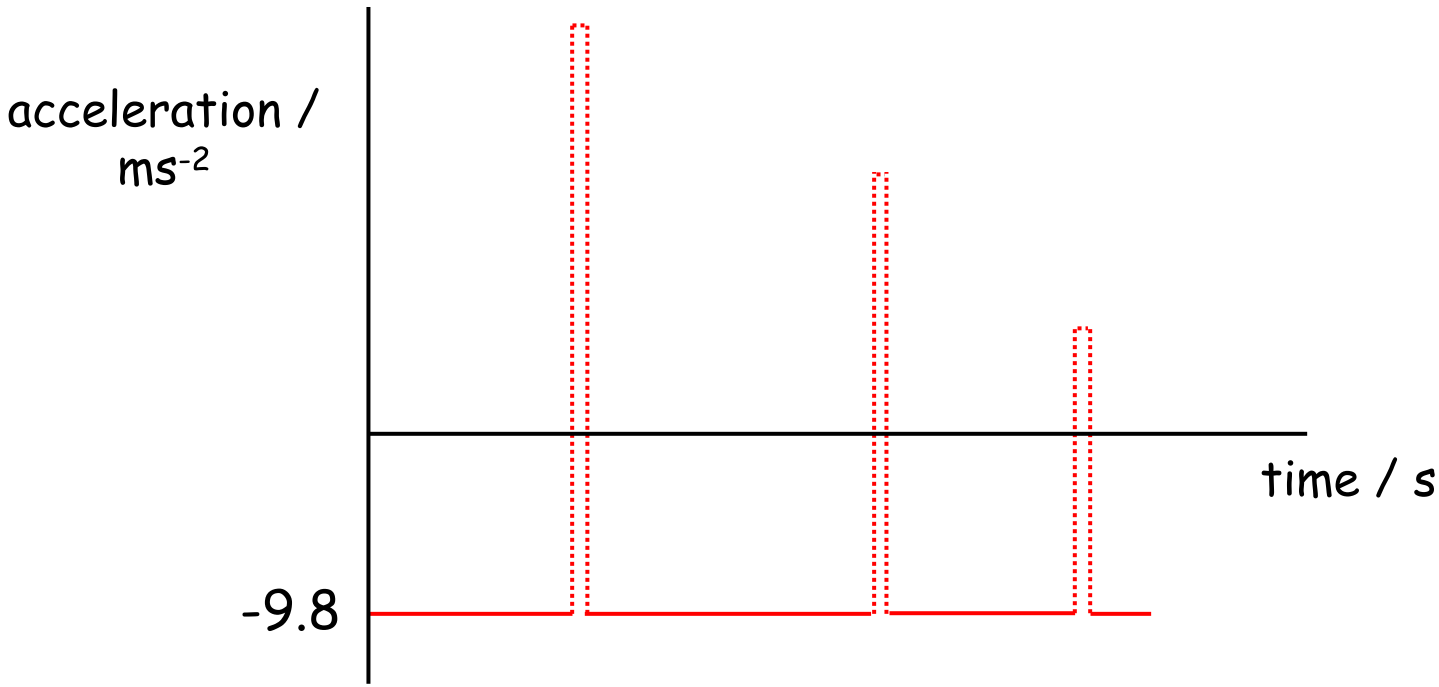
Bouncing Ball (With Energy Loss)

A ball is dropped from a height to the ground.

Kinetic energy is lost with each bounce.

The velocity - time and acceleration - time graphs are as follows:




acceleration / ms^{-2}
time / s
-9.8

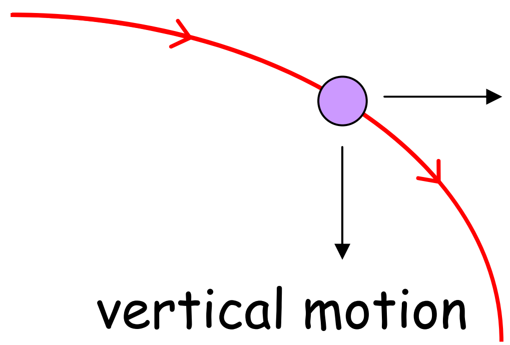
<u>Worksheet – Kinematics Problems</u>

More Graphs

Q1, Q2, Q3 & Q5

Projectiles 1

An object projected sideways through the air will follow a curved trajectory.



horizontal motion
(steady speed)

$$V_H = \frac{D_H}{t}$$

vertical motion

accelerates downwards at -9.8 ms^{-2}

The horizontal and vertical motions should be treated separately.

Time is the only quantity common to both.

At any point in its trajectory, the velocity of a projectile has 2 components.

- one vertical, V_V
- the other horizontal, V_H

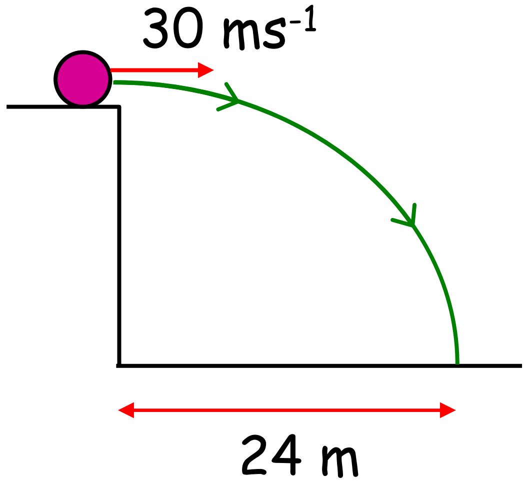
The resultant velocity is found drawing a vector diagram and adding the vectors together.

Example

A ball is kicked horizontally off an embankment, with a velocity of 30 ms^{-1}.

It lands 24 m from the base of the embankment.

(a) Calculate how long the ball was in flight.



$$V_H = \frac{D_H}{t}$$

$$30 = \frac{24}{t}$$

$$t = \frac{24}{30}$$

$$t = 0.8\ s$$

common to horizontal and vertical motions

(b) Calculate the horizontal velocity just before hitting the ground.

Horizontal

$u = 30\ ms^{-1}$

$s = 24\ m$

$t = 0.8\ s$

$a = 0\ ms^{-2}$

travels horizontally at steady speed – no acceleration horizontally

Vertical

acted upon by gravity

$a = -9.8\ ms^{-2}$

$t = 0.8\ s$

$u = 0\ ms^{-1}$

not initially falling down, so speed of zero in vertical direction

Horizontal

$$v = u + at$$

$$= 30 + (0 \times 0.8)$$

$$v = 30\ ms^{-1}$$

(c) Calculate the vertical velocity just before hitting the ground.

Vertical

$$v = u + at$$
$$= 0 + (-9.8 \times 0.8)$$
$$v = -7.84\ ms^{-1}$$

means 7.84 ms^{-1} downwards

(d) How high is the embankment?

Vertical

$a = -9.8\ ms^{-2}$

$t = 0.8\ s$

$u = 0\ ms^{-1}$

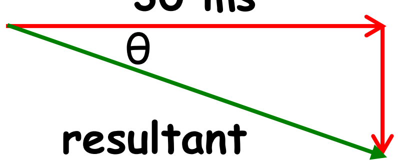
$v = -7.84\ ms^{-1}$

$$s = ut + \frac{1}{2}at^2$$
$$= (0 \times 0.8) + \frac{1}{2}(-9.8 \times 0.8^2)$$
$$s = -3.14\ m$$

means ball fell through distance of 3.14 m

so height of the embankment is 3.14 m

(e) Calculate the resultant velocity of the ball, just before hitting the ground.



Size

By Pythagoras:

$$a^2 = b^2 + c^2$$

$$(\text{resultant velocity})^2 = 30^2 + (-7.84)^2$$

$$= 900 + 61.5$$

$$\text{resultant velocity} = \sqrt{961.5}$$

$$= \underline{\underline{31\,\text{ms}^{-1}}}$$

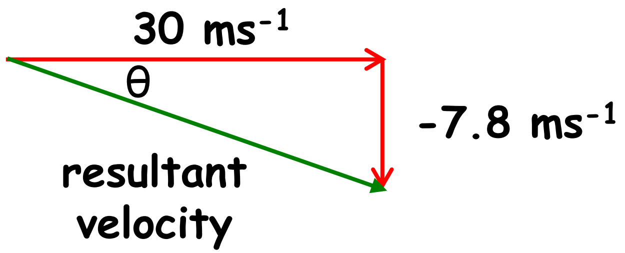
Direction

$$\cos\theta = \frac{adj}{hyp}$$

$$\cos\theta = \frac{30}{31}$$

$$\theta = \cos^{-1}(0.97)$$

$$\theta = 14.6^\circ$$



$$\text{resultant velocity} = 31\,\text{ms}^{-1}\ \text{at angle of}\ 14.6^\circ\ \text{below horizon}$$

Q1. A ball is kicked off a cliff with a horizontal speed of 16 ms^{-1}.

The ball hits the ground 2.2 s later.

(a) Calculate the height of the cliff. 23.7 m

(b) Calculate the distance between the foot of the cliff and where the ball lands. 35.2 m

(c) Calculate the vertical component of the balls velocity just before it hits the ground. 21.6 ms^{-1}

(d) Calculate the balls velocity as it hits the ground.

26.9 ms^{-1} at angle of 53.5° below horizon

You may want to draw a diagram to help you get started !!!

Q2. A ball is kicked off a cliff with a horizontal speed of 22 ms^{-1}. the ball hits the ground 1.5 s later.

(a) Calculate the height of the cliff. 11 m

(b) Calculate the horizontal distance from the foot of the cliff, to where the ball lands. 33 m

(c) Calculate the vertical component of the balls velocity as it hits the ground. 14.7 ms^{-1}

(d) Calculate the balls actual velocity as it hits the ground.

26.5 ms^{-1} at angle of 34° below horizon

You may want to draw a diagram to help you get started !!!

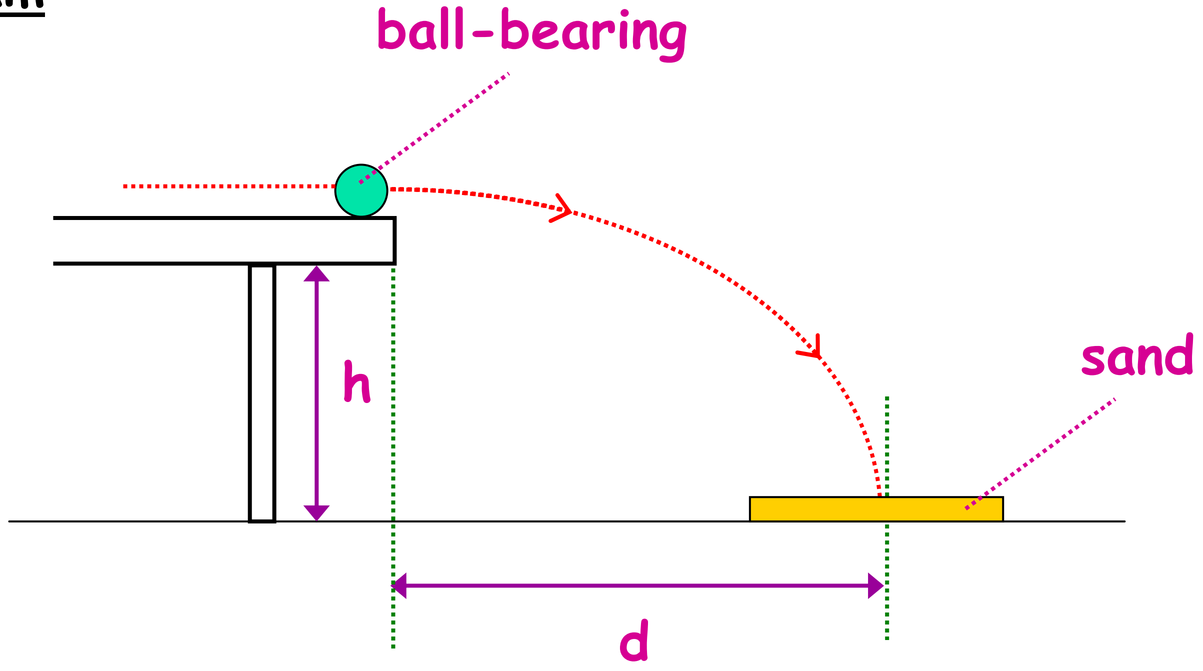
Worksheet – Kinematics Problems

Projectiles

Q1 – Q8

Does Projectile Theory Work?

<u>Diagram</u>



Measurements

Horizontal Velocity — measure distance ball-bearing travels along desk and divide by time taken

Vertical Displacement — measure height of desk from floor

Calculation

Calculate the time of flight.

Vertical

$s = \_\_\_\_\_ \text{ m}$

$a = -9.8 \text{ ms}^{-2}$

$u = 0 \text{ ms}^{-1}$

$t = ?$

$$s = ut + \frac{1}{2}at^2$$

Now calculate the horizontal displacement.

Horizontal

$v_H = \_\_\_\_ \; ms^{-1}$

$s = v_H \times t$

$t = \_\_\_\_\_ \; s$

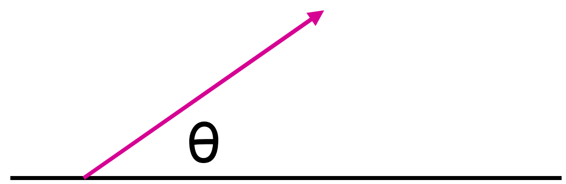
$s = ?$

Experimentally

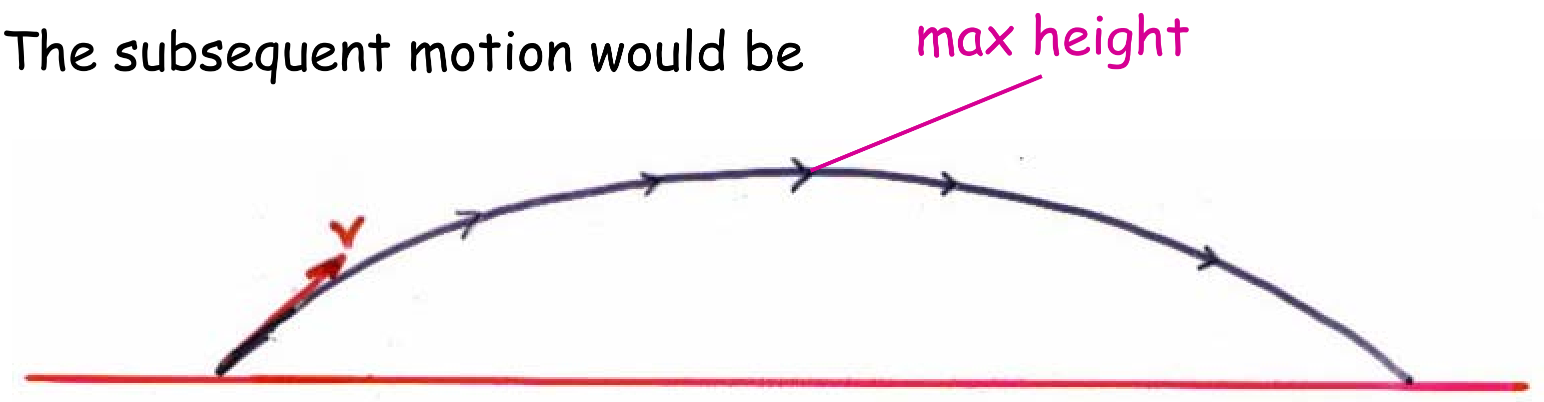
The horizontal displacement was measured experimentally using a metre stick to be ______ m.

Projectiles 2

A projectile does not need to be an object falling, but may be an object fired at angle to the horizontal.



The subsequent motion would be



If air resistance is ignored, the trajectory has an axis of symmetry about the mid point (maximum height).

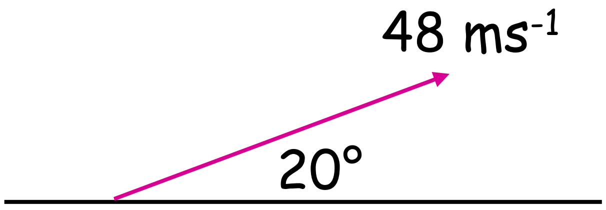
So the time taken to reach the maximum height is the same as the time taken to fall back to the ground.

Various calculations can be made, but firstly, the initial velocity must be split into its horizontal and vertical components.

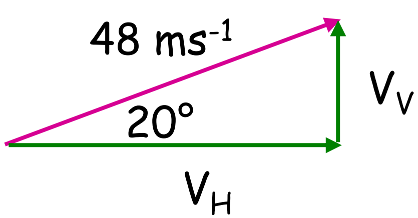
Horizontal	Vertical
$a = 0 \text{ ms}^{-2}$	$a = 9.8 \text{ ms}^{-2}$

Example 1

A golf ball is hit off the tee at 48 ms^{-1} at angle of 20° to the horizontal.



Calculate the horizontal and vertical components of the initial velocity.



Horizontal

$$\cos\theta = \frac{adj}{hyp}$$

$$\cos 20 = \frac{v_H}{48}$$

$$v_H = 48 \times \cos 20$$

$$v_H = 45.1\ ms^{-1}$$

Vertical

$$\sin\theta = \frac{opp}{hyp}$$

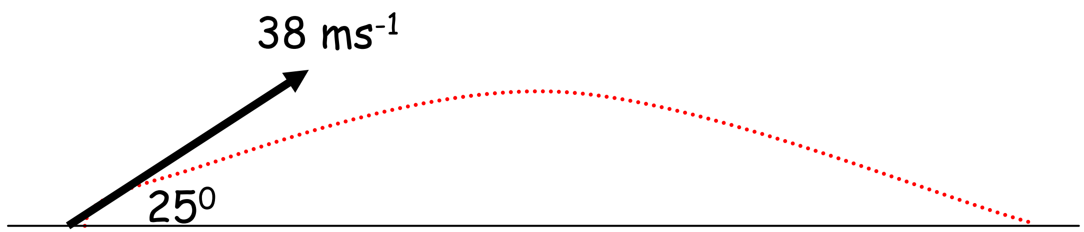
$$\sin 20 = \frac{v_V}{48}$$

$$v_V = 48 \times \sin 20$$

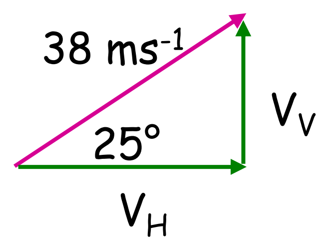
$$v_V = 16.4\ ms^{-1}$$

Example 2

An arrow is projected into the air with a velocity of 38 ms^{-1} at an angle of 25° to the horizontal.



(a) Calculate the horizontal and vertical components of the initial velocity.



Horizontal

$$\cos\theta = \frac{adj}{hyp}$$

$$\cos 25 = \frac{v_H}{38}$$

$$v_H = 38 \times \cos 25$$

$$v_H = 34.4\ ms^{-1}$$

Vertical

$$\sin\theta = \frac{adj}{hyp}$$

$$\sin 25 = \frac{v_V}{38}$$

$$v_V = 38 \times \sin 25$$

$$v_V = 16.1\ ms^{-1}$$

(b) Calculate the arrow's maximum height.

Vertical

$a = -9.8\,ms^{-2}$

$u = 16.1\,ms^{-1}$

$v = 0\,ms^{-1}$

$s = ?$

$$v^2 = u^2 + 2as$$

$$0^2 = 16.1^2 + (2 \times -9.8 \times s)$$

$$0 = 259.21 - 19.6\,s$$

$$19.6\,s = 259.21$$

$$s = 13.2\,m$$

(c) Calculate the time taken for the arrow to reach its maximum height.

Vertical

$a = -9.8 \, ms^{-2}$

$u = 16.1 \, ms^{-1}$

$v = 0 \, ms^{-1}$

$s = 13.2 \, m$

$t = ?$

$$v = u + at$$

$$0 = 16.1 + (-9.8 \times t)$$

$$9.8\,t = 16.1$$

$$t = 1.64 \, s$$

(d) Calculate the total time of the arrows flight.

$$\text{total time} = \text{time up} + \text{time down}$$

$$= 1.64 + 1.64$$

$$= 3.28 \, s$$

(e) Calculate the horizontal distance travelled by the arrow until impact with the ground.

Horizontal

$u = 34.4\ ms^{-1}$

$a = 0\ ms^{-2}$

$t = 3.28\ s$

$s = ?$

$$s = ut + \frac{1}{2}at^2$$

$$= (34.4 \times 3.28) + \left(\frac{1}{2} \times 0 \times 3.28^2\right)$$

$$s = 112.8m$$

(f) Calculate the arrow's velocity 0.5 s after being fired.

Firstly calculate the vertical component of velocity (horizontal component is constant, since a = 0 ms^{-2})

Vertical

$a = -9.8\ ms^{-2}$

$u = 16.1\ ms^{-1}$

$t = 0.5\ s$

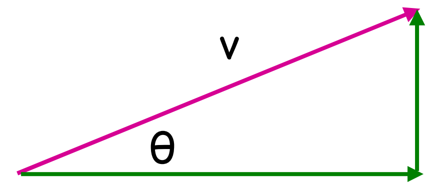
$v = ?$

$$v = u + at$$
$$= 16.1 + (-9.8 \times 0.5)$$
$$= 16.1 - 4.9$$
$$v = 11.2\ ms^{-1}$$

Now calculate the actual velocity after combining the vertical and horizontal components of the velocity after 0.5 s.

Size

By Pythagoras:



$$a^2 = b^2 + c^2$$

$$(\text{velocity})^2 = 34.4^2 + 11.2^2$$

$$= 1{,}183.36 + 125.44$$

$$\text{velocity} = \sqrt{1{,}308.8}$$

$$= 36.2\ \text{ms}^{-1}$$

Direction

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan\theta = \frac{11.2}{34.4}$$

$$\theta = \tan^{-1}(0.326)$$

$$\theta = 18^\circ$$

Velocity of the arrow after 0.5 s is:

36.2 ms^{-1} at angle of 18° above the horizon

Q1. A shell is fired from a gun with a velocity of 72 ms^{-1} at an angle of 60° to the horizontal.

(a) Calculate the horizontal and vertical components of the initial velocity. V_H = 36 ms^{-1} V_V = 62.4 ms^{-1}

(b) Calculate the maximum height reached. 199 m

(c) Calculate the time taken for the shell to reach it's maximum height. 6.4 s

(d) Calculate the total time of flight. 12.8 s

(e) Calculate the horizontal range of the shell. 458 m

(f) Calculate the shells velocity after 2.3 s.

53.7 ms^{-1} at angle of 48° above the horizon

Q2. An arrow is fired with a velocity of 50 ms^{-1} at an angle of 30° to the ground.

(a) Calculate the time taken for the arrow to reach its maximum height. 2.55 s

(b) Calculate the maximum height reached by the arrow. 31.89 m

(c) Calculate the time the arrow is in flight. 5.1 s

(d) Calculate how far away from the firing point the arrow will land. 5.1 s

(e) Calculate the actual velocity of the arrow 1s after it is fired. 45.89 ms^{-1} at angle of 19.3° above horizon

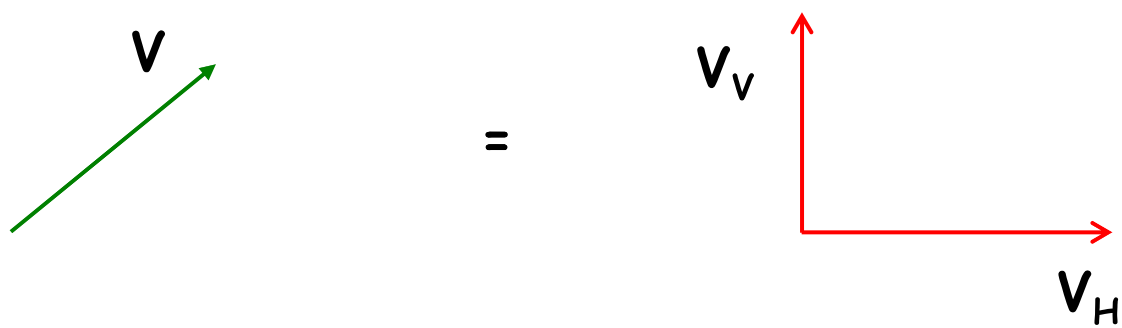
Worksheet – Kinematics Problems

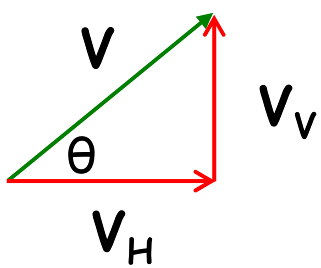
Projectiles

Q9 - Q15

Horizontal and Vertical Components

It some cases it may be useful to "break-up" or resolve a vector into its rectangular components.





$$\sin\theta = \frac{V_V}{V}$$

$$V_V = V\sin\theta$$

$$\cos\theta = \frac{V_H}{V}$$

$$V_H = V\cos\theta$$