Higher Physics – Unit 1

1.4 – Momentum and Impulse

Momentum

The **momentum** of an object is the **product** of it's **mass and velocity**.

The **MOMENTUM** of an object is calculated by:

$$p = m\,v$$

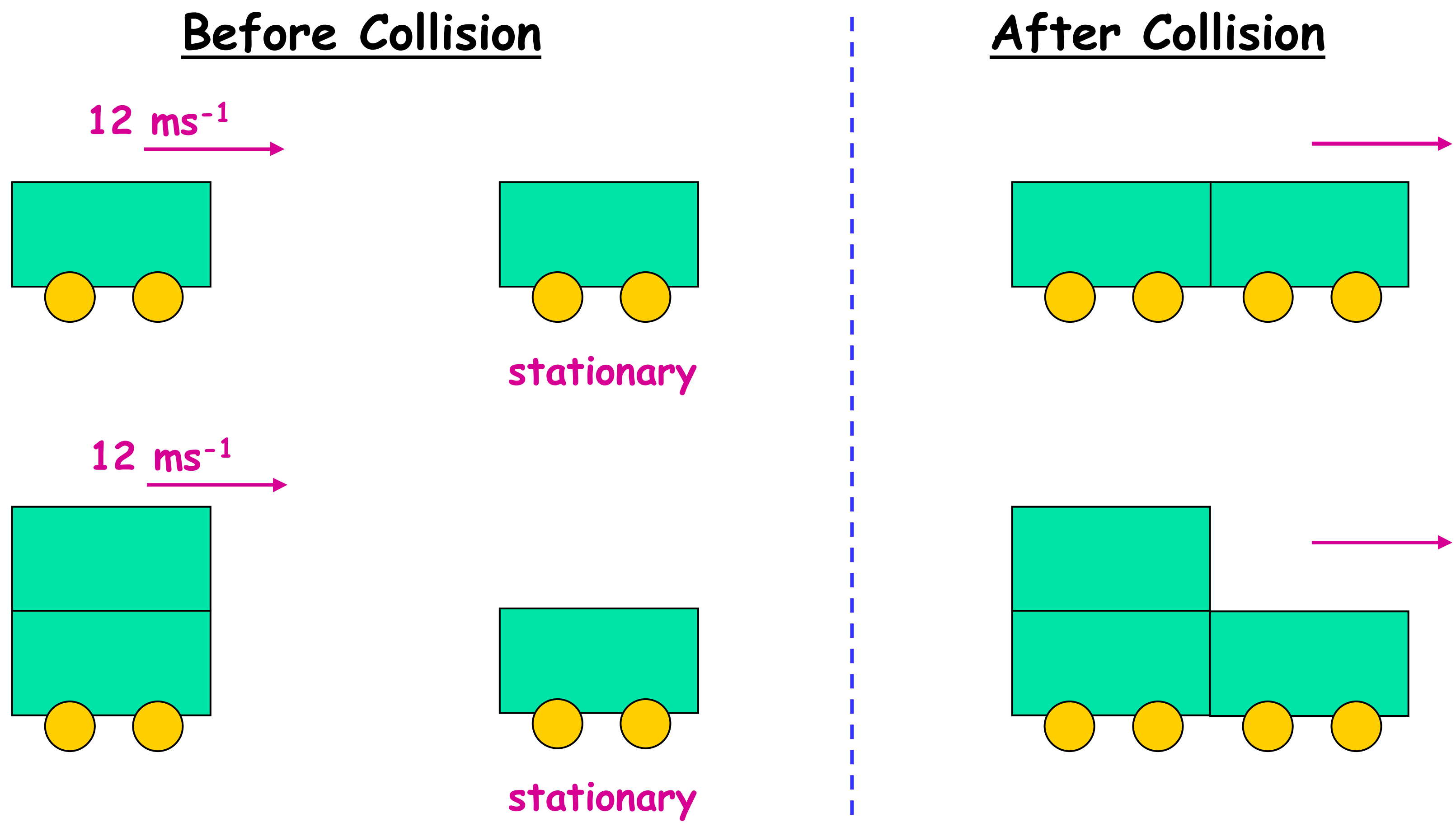
velocity (ms^{-1})

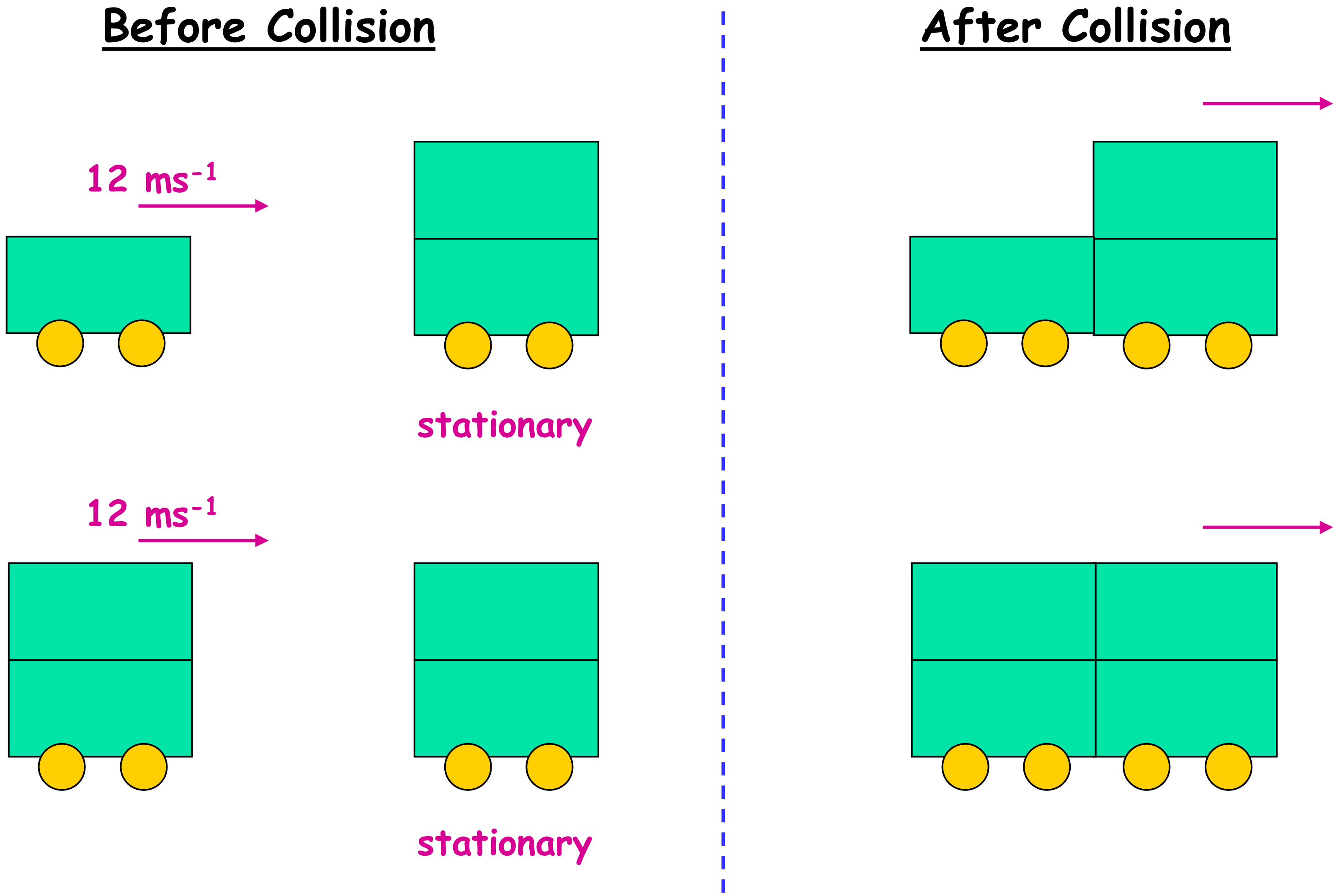
momentum ($kg\ ms^{-1}$)

mass (kg)

Momentum is a **vector** (has both magnitude and direction).

Collisions


Before Collision
After Collision
12 ms-1
stationary
12 ms-1
stationary


Before Collision
After Collision
12 ms-1
stationary
12 ms-1
stationary

Results

Before Collision

Mass	Velocity
1	12
2	12
1	12
2	12

After Collision

Mass	Velocity
2	6
3	8
3	4
4	6

Conclusion

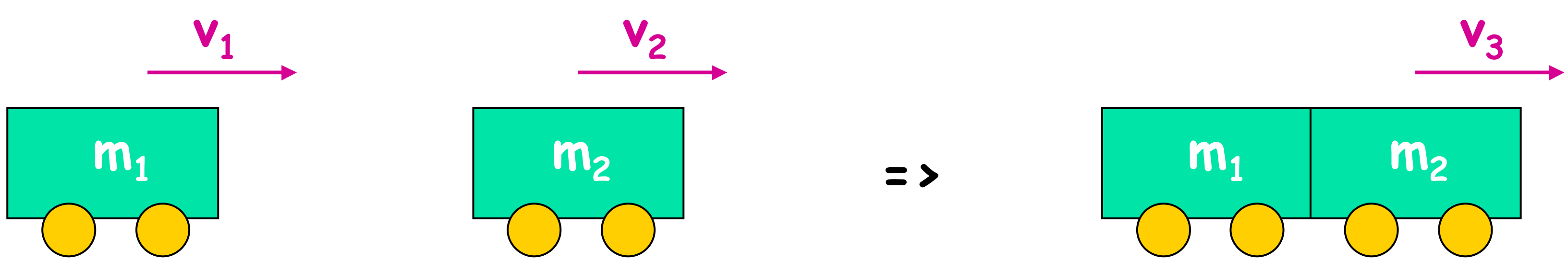
BEFORE **AFTER**

mass × velocity = mass × velocity

MOMENTUM IS CONSERVED

Momentum and Collisions

The **total momentum** is **CONSERVED** in collisions provided there are no external forces (e.g. friction).



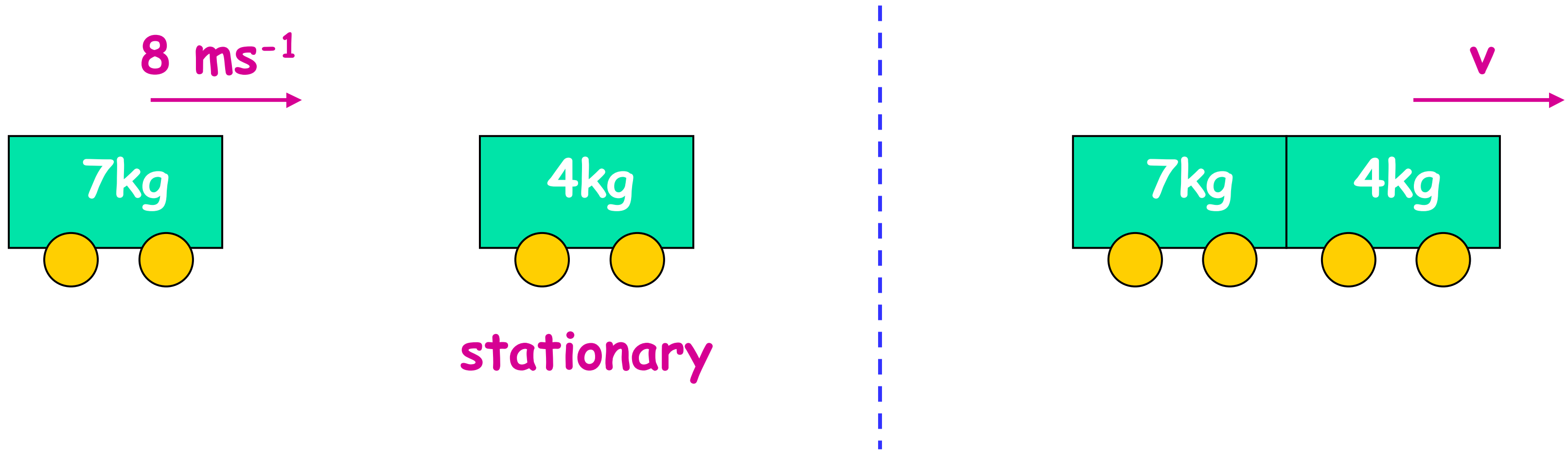
total momentum before = total momentum after

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_3$$

Example 1

A 7kg mass travelling at 8 ms^{-1} collides and sticks to a stationary 4 kg mass.

Calculate the velocity just after impact.



total momentum before = total momentum after

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_3$$

$$(7 \times 8) + (4 \times 0) = 11\,v$$

$$11\,v = 56$$

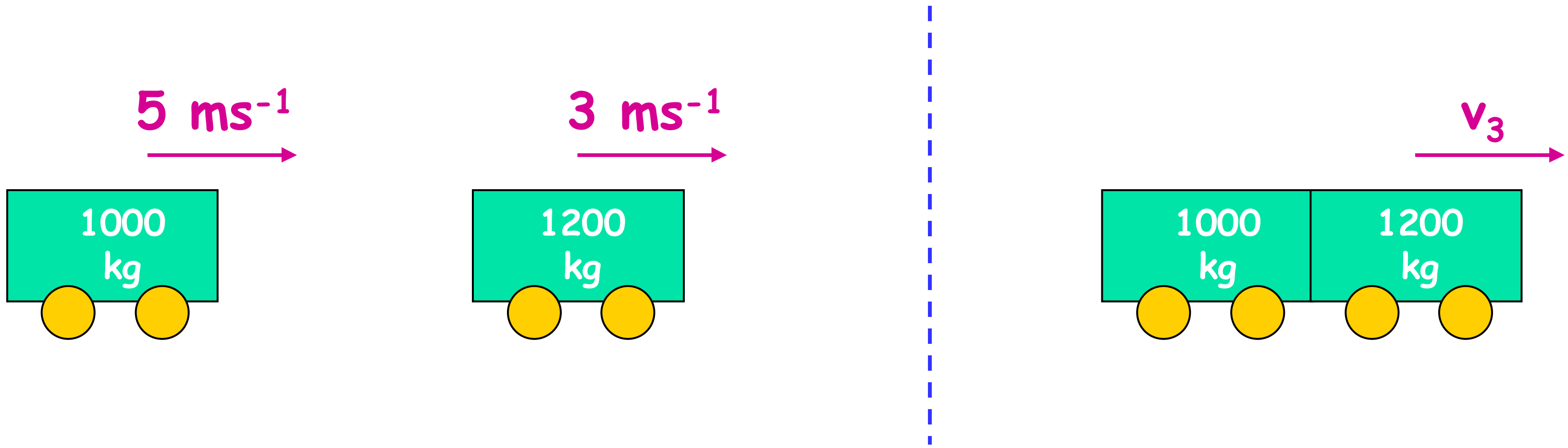
$$v = 5.1\,ms^{-1} \text{ to the right}$$

Example 2

A car of mass 1,000 kg is travelling at 5 ms^{-1}.

It collides and joins with a 1,200 kg car travelling at 3 ms^{-1}.

Calculate the velocity of the cars just after impact.



total momentum before = total momentum after

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_3$$

$$(1000 \times 5) + (1200 \times 3) = 2200\, v$$

$$2200\, v = 8600$$

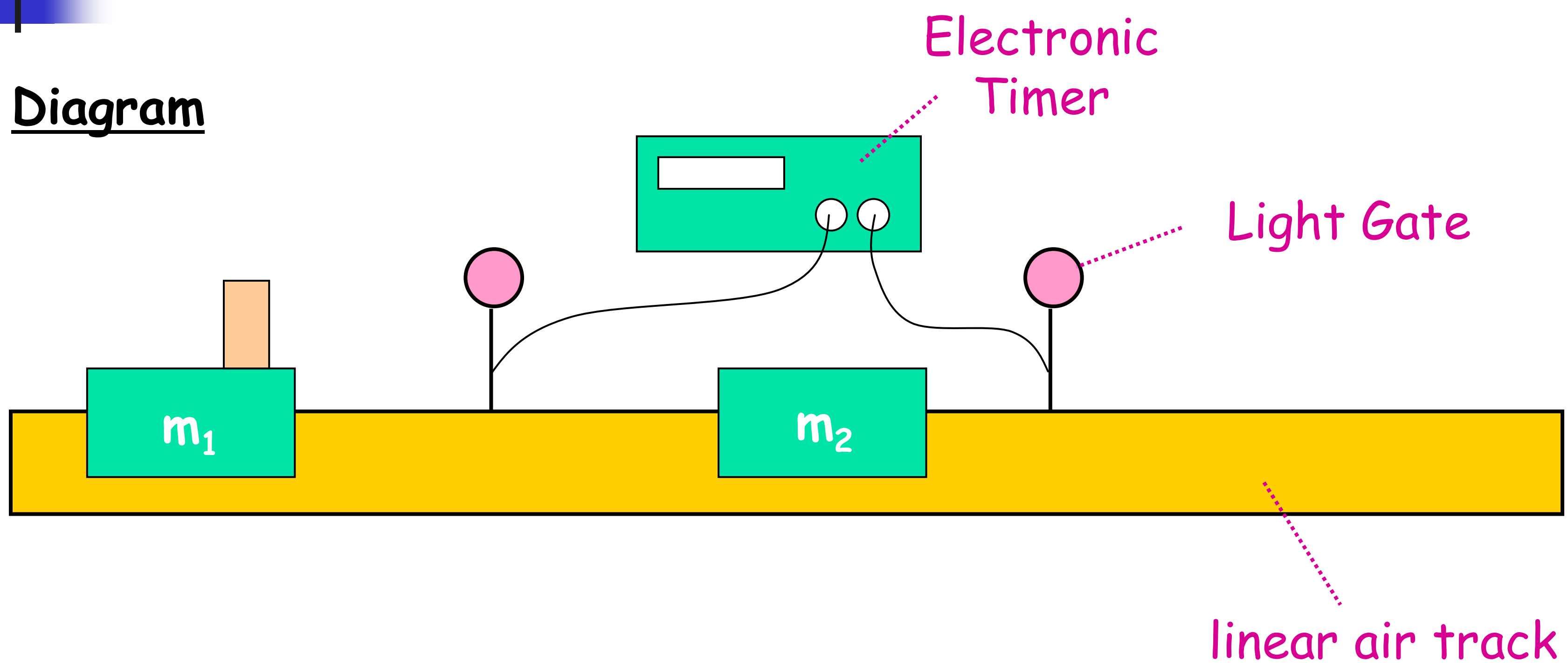
$$v = 3.9\ ms^{-1} \text{ to the right}$$

Worksheet – Momentum and Collisions

Q1 – Q6

Is Momentum Conserved?

Diagram



Procedure

Vehicle 1 is sprung along the air track.

It breaks the first light gate and a velocity is given.

It collides and sticks to the second vehicle.

They both move together and break the second light gate, giving a second velocity.

Results

m_1 = ____ kg m_2 = ____ kg m_3 = ____ kg

v_1 = _____ ms^{-1} v_2 = _____ ms^{-1} v_3 = _____ ms^{-1}

$$\text{total momentum before} = m_1 v_1 + m_2 v_2$$

$$= \_\_\_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_\_\_$$

$$= \_\_\_\_\_\_\_\_\_\_ \text{ kg ms}^{-1}$$

$$\text{total momentum after} = m_3 v_3$$

$$= \_\_\_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_\_\_$$

$$= \_\_\_\_\_\_\_\_\_\_ \text{ kg ms}^{-1}$$

Conclusion

Momentum is CONSERVED.

total momentum before = total momentum after

Elastic Collisions

KINETIC ENERGY CONSERVED

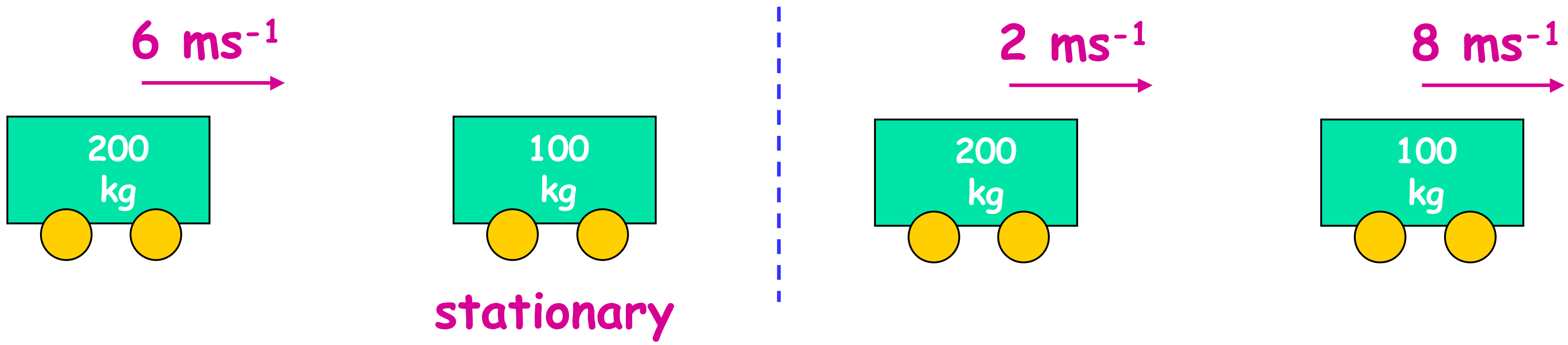
MOMENTUM CONSERVED

Example 1

A 200 kg vehicle is travelling at 6 ms^{-1} when it collides with a stationary 100 kg vehicle.

After the collision, the 200 kg vehicle moves off at 2 ms^{-1} and the 100 kg vehicle at 8 ms^{-1}.

Show the collision is elastic.



Momentum

Before

$m_1 v_1 + m_2 v_2$

$(200 \times 6) + (100 \times 0)$

1200 kg ms^{-1}

After

$m_1 v_3 + m_2 v_4$

$(200 \times 2) + (100 \times 8)$

1200 kg ms^{-1}

Momentum has been conserved.

Kinetic Energy

Before

$$E_K = \frac{1}{2} m v^2$$

$$= \left(\frac{1}{2} \times 200 \times 6^2\right) + \left(\frac{1}{2} \times 100 \times 0^2\right)$$

$$E_K = 3600 \text{ J}$$

After

$$E_K = \frac{1}{2} m v^2$$

$$= \left(\frac{1}{2} \times 200 \times 2^2\right) + \left(\frac{1}{2} \times 100 \times 8^2\right)$$

$$E_K = 3600 \text{ J}$$

Kinetic Energy has been conserved.

As momentum and kinetic energy are conserved, ELASTIC collision.

Inelastic Collisions

Kinetic Energy NOT Conserved

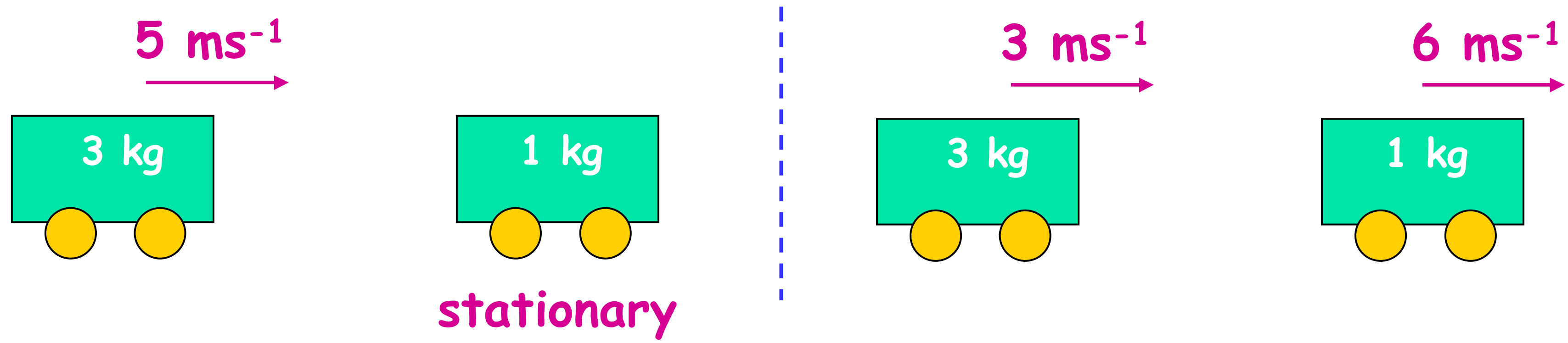
MOMENTUM CONSERVED

Example 1

A trolley of mass 3 kg is travelling at 5 ms^{-1} when it collides with a stationary 1 kg trolley.

Afterwards, they move off at 3 ms^{-1} and 6 ms^{-1} respectively.

Show that this collision is inelastic.



Momentum

Before

$m_1 v_1 + m_2 v_2$

$(3 \times 5) + (1 \times 0)$

15 kg ms^{-1}

After

$m_1 v_3 + m_2 v_4$

$(3 \times 3) + (1 \times 6)$

15 kg ms^{-1}

Momentum has been conserved.

Kinetic Energy

Before

$$E_K = \frac{1}{2} m v^2$$

$$= \left(\frac{1}{2} \times 3 \times 5^2\right) + \left(\frac{1}{2} \times 1 \times 0^2\right)$$

$$E_K = 37.5 \text{ J}$$

After

$$E_K = \frac{1}{2} m v^2$$

$$= \left(\frac{1}{2} \times 3 \times 3^2\right) + \left(\frac{1}{2} \times 1 \times 6^2\right)$$

$$E_K = 31.5 \text{ J}$$

Kinetic Energy has NOT been conserved.

As momentum is conserved and kinetic is not, INELASTIC collision.

TOTAL ENERGY is always CONSERVED

total energy = kinetic energy + heat energy + sound energy

Note

In reality, most collisions are inelastic.

Some of the kinetic energy is converted to heat and sound energy on impact.

Worksheet – Elastic and Inelastic Collisions

Q1 - Q3

Head On Collisions

Head on collisions involve objects travelling in **opposite directions**.

One direction is **POSITIVE**, the other then has to be **NEGATIVE**.

Example 1

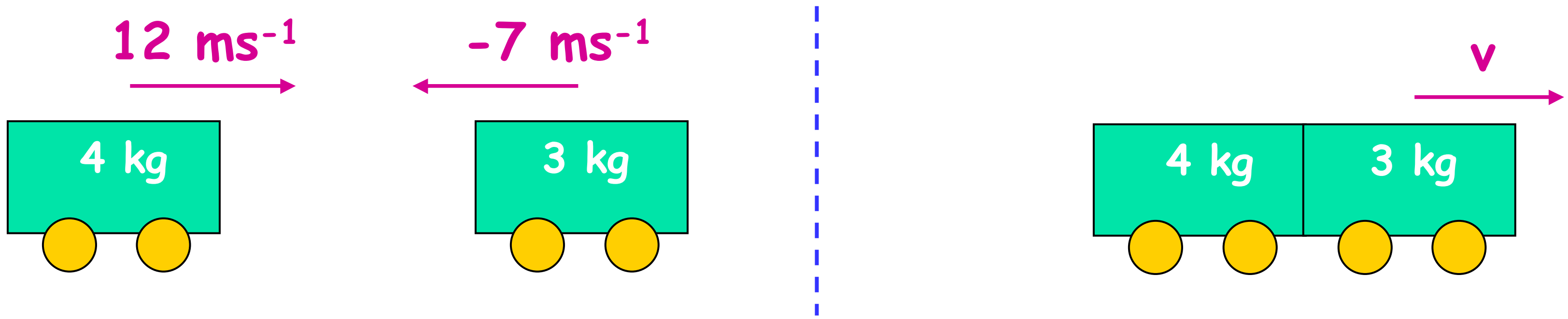
A 4 kg object travels at 12 ms^{-1} and collides head on with a 3 kg object travelling with a speed of 7 ms^{-1}.

After the collision, they both move off together.

(a) calculate the velocity of the objects just after impact.

(b) determine whether the collision is elastic or inelastic.

(a)



total momentum before = total momentum after

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$$

$$(4 \times 12) + (3 \times (-7)) = (4 + 3) v$$

$$7 v = 48 - 21$$

$$7 v = 27$$

$$v = 3.86 \text{ ms}^{-1} \text{ to the right}$$

(b) Kinetic Energy

Before

$$E_K = \frac{1}{2} m v^2$$

$$= \left(\frac{1}{2} \times 4 \times 12^2 \right) + \left(\frac{1}{2} \times 3 \times (-7)^2 \right)$$

$$= 288 + 73.5$$

$$E_K = 361.5 \text{ J}$$

After

$$E_K = \frac{1}{2} m v^2$$

$$= \left(\frac{1}{2} \times 7 \times 3.86^2 \right)$$

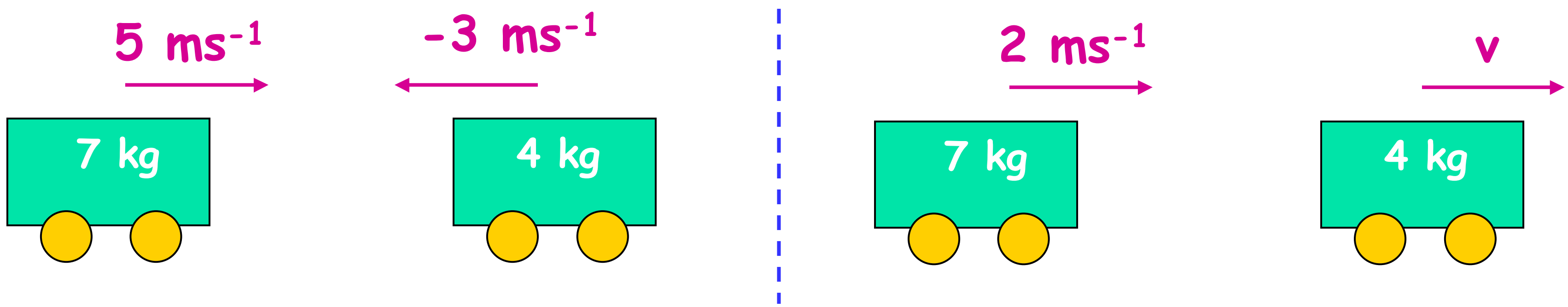
$$E_K = 52.1 \text{ J}$$

Kinetic Energy has NOT been conserved.

INELASTIC collision.

Example 2

Two objects collide as shown.



(a) Calculate the velocity at which the 4 kg object moves, just after impact.

(b) Determine whether the collision is elastic or inelastic.

(a)

total momentum before = total momentum after

$$m_1 v_1 + m_2 v_2 = m_1 v_3 + m_2 v_4$$

$$(7 \times 5) + (4 \times (-3)) = (7 \times 2) + (4 \times v)$$

$$35 - 12 = 14 + 4v$$

$$23 - 14 = 4v$$

$$4v = 9$$

$$v = 2.25\ ms^{-1} \text{ to the right}$$

(b) **Kinetic Energy**

Before

$$E_K = \frac{1}{2} m v^2$$

$$= \left(\frac{1}{2} \times 7 \times 5^2\right) + \left(\frac{1}{2} \times 4 \times (-3)^2\right)$$

$$= 87.5 + 18$$

$$E_K = 105.5 \text{ J}$$

After

$$E_K = \frac{1}{2} m v^2$$

$$= \left(\frac{1}{2} \times 7 \times 2^2\right) + \left(\frac{1}{2} \times 4 \times 2.25^2\right)$$

$$= 14 + 10.13$$

$$E_K = 24.13 \text{ J}$$

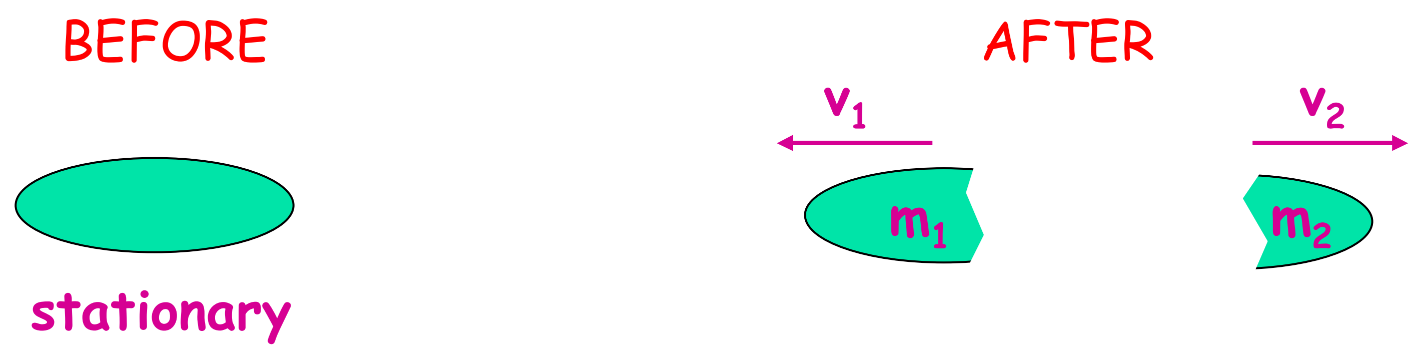
Kinetic Energy has NOT been conserved.

INELASTIC collision.

Explosions

In all explosions:

MOMENTUM CONSERVED



total momentum before = total momentum after

$$m v = m_1 v_1 + m_2 v_2$$

$$0 = m_1 (-v_1) + m_2 v_2$$

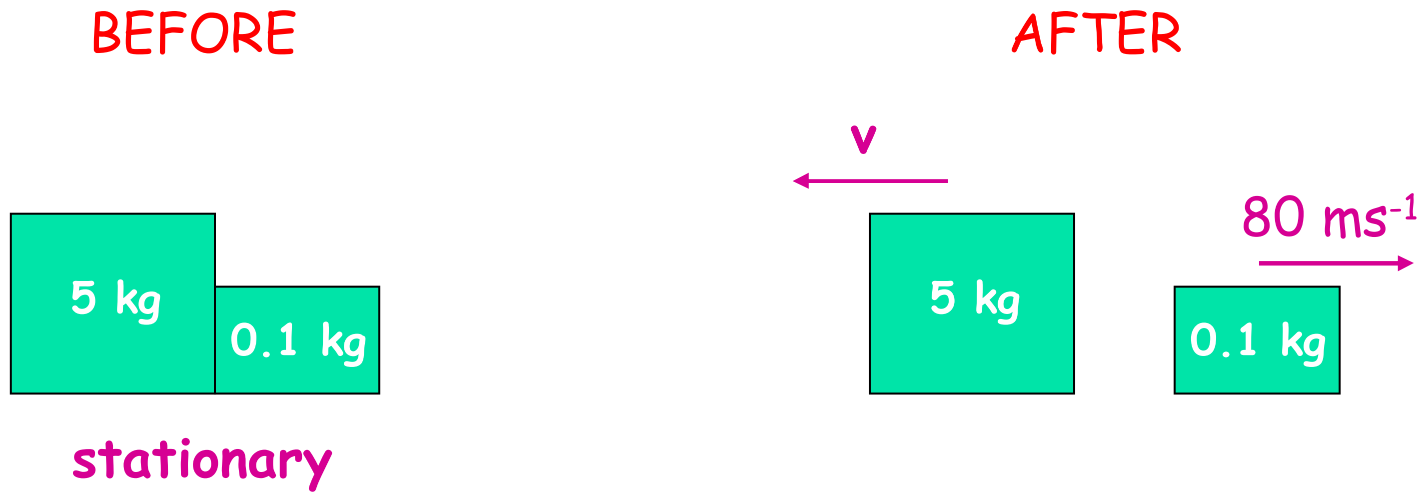
$$0 = -m_1 v_1 + m_2 v_2$$

Example 1

A 5 kg gun fires a 0.1 kg shell at 80 ms^{-1}.

The gun recoils after firing the shell.

Calculate the recoil speed of the gun.



total momentum before = total momentum after

$$m v = m_1 v_1 + m_2 v_2$$

$$0 = 5(v) + (0.1 \times 80)$$

$$-5v = 8$$

$$v = -1.6\ ms^{-1} \quad \text{backwards}$$

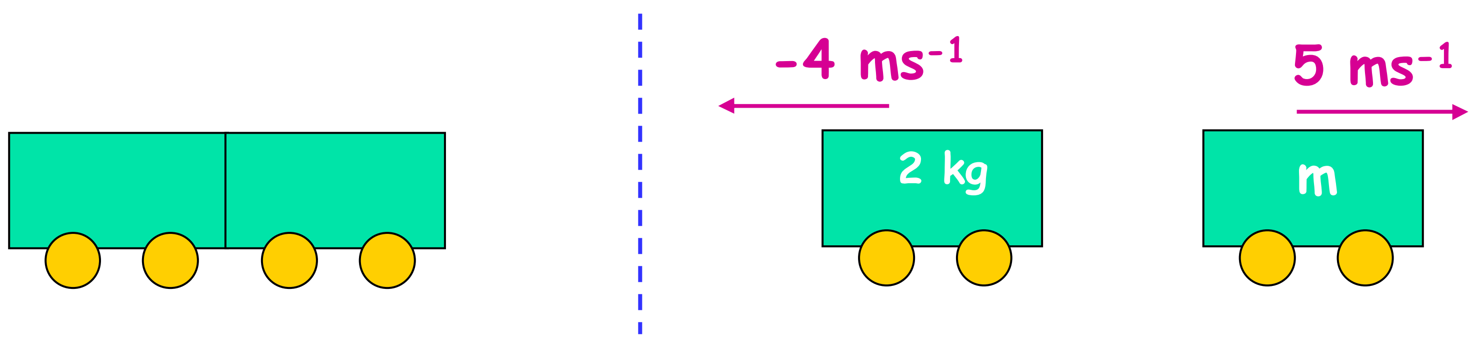
Example 2

Two trolleys initially at rest and touching, fly apart when the plunger is released.

One trolley with a mass of 2 kg moves off with a speed of 4 ms^{-1}.

The other trolley moves off in the opposite direction with a speed of 5 ms^{-1}.

Calculate the mass of this trolley.



total momentum before = total momentum after

$$m v = m_1 v_1 + m_2 v_2$$

$$0 = 2 \times (-4) + (m \times 5)$$

$$5m = 8$$

$$\underline{\underline{m = 1.6 \text{ kg}}}$$

Worksheet – Head On Collisions and Explosions

Q1 – Q8

Impulse & Change in Momentum

Consider the following equations:

$$F = m\,a$$

$$a = \frac{v - u}{t}$$

Combining these equations:

$$F = m\,a$$

$$F = \frac{m\,(v - u)}{t}$$

$$F\,t = mv - mu$$

IMPULSE (F t) = CHANGE IN MOMENTUM (mv – mu)

$$F\,t = mv - mu$$

- force (N)
- time (s)
- mass (kg)
- final velocity (ms^{-1})
- initial velocity (ms^{-1})

Impulse

IMPULSE is the product of the **FORCE** and the **TIME** during which it acts.

$$\text{impulse} = F\,t$$

***** NOT ON DATA SHEET *****

The units of impulse are **N s** (Newton Seconds).

Impulse is a **vector** quantity.

Change In Momentum

The **change in momentum** is the difference in momentum from when the object is moving at its **initial speed** until it reaches its **final speed**.

$$\text{change in momentum} = mv - mu$$

***** NOT ON DATA SHEET *****

The unit of change in momentum is **kg ms^{-1}**.

Impulse and Change in Momentum

Impulse and **change in momentum** are **equal** to each other.

So if you know **impulse** is **2 Ns**, **without any further calculation**, you can state the **change in momentum** to be **2 kg ms^{-1}**.

$$Ft = mv - mu$$

IMPULSE = CHANGE IN MOMENTUM

Example 1

A golf ball of mass 50 g is hit off the tee at 30 ms^{-1}.

The time of contact between club and ball is 25 ms (milliseconds).

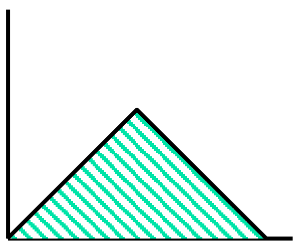
Calculate the average force exerted on the ball.

$$m = 50\ g$$
$$= 0.05\ kg$$
$$u = 0\ ms^{-1}$$
$$v = 30\ ms^{-1}$$
$$t = 25\ ms$$
$$= 25 \times 10^{-3}\ s$$

$$F\,t = mv - mu$$
$$F \times (25 \times 10^{-3}) = (0.05 \times 30) - (0.05 \times 0)$$
$$F = \frac{1.5}{25 \times 10^{-3}}$$
$$F = 60\ N$$

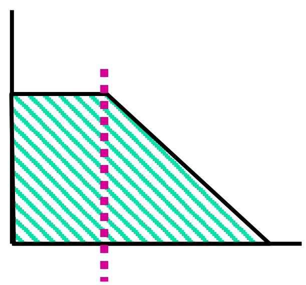
Impulse, Force and Time

impulse = area under force-time graph



impulse = area under graph

$$= \frac{1}{2} \times b \times h$$

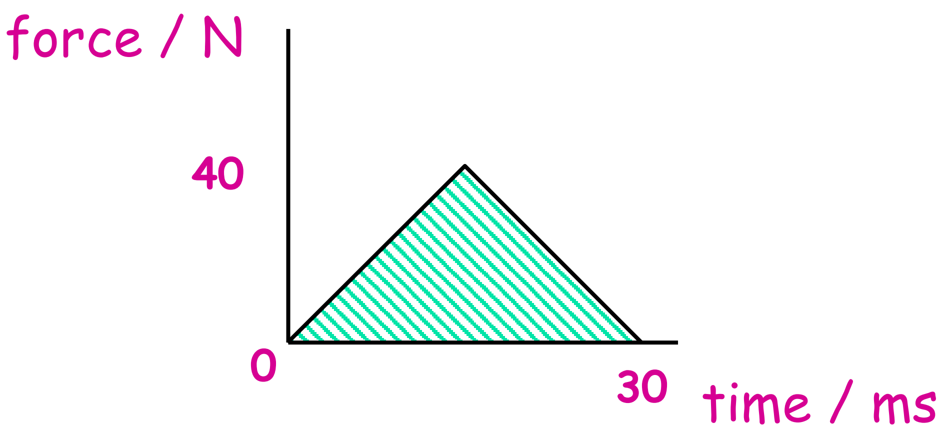


impulse = area under graph

$$= l \times b + \frac{1}{2} \times b \times h$$

Example 1

A 50 g golf ball is hit off the tee by a force which varies with time as shown.



Calculate the speed of the golf ball off the tee.

$$\text{impulse} = \text{area under graph}$$
$$= \frac{1}{2} \times b \times h$$
$$= \frac{1}{2} \times (30 \times 10^{-3}) \times 40$$
$$= 0.6 \text{ N s}$$

$$\text{impulse} = mv - mu$$
$$0.6 = 0.05v - 0.05 \times 0$$
$$0.05v = 0.6$$
$$v = \frac{0.6}{0.05}$$
$$v = 12 \text{ ms}^{-1}$$

Change In Momentum

Air Bags

A passenger in a car involved in a collision will experience a force which will bring him to a stop.

NO Air Bag

- Head hits hard object eg. steering wheel
- In contact for a **short time**
- **Large force** involved
- **Lots of Damage**

AIR BAG

- Head hits air bag
- In contact for a **longer time**
- **Smaller force** involved
- **Less damage**

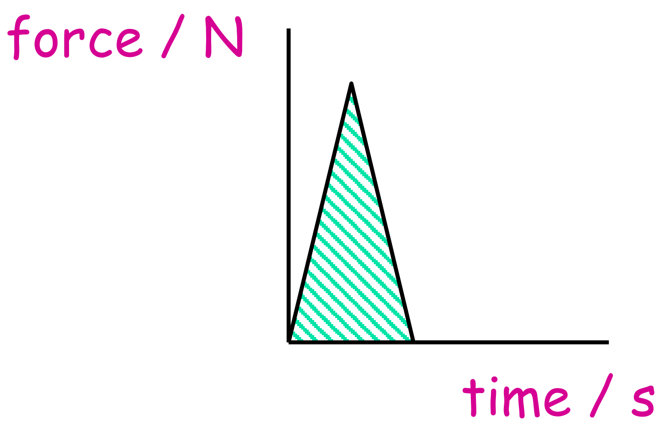
In **both cases** the **change in momentum** and therefore the impulse are the **same**.

However, the **force-time graphs** will **differ in shape** although the **area** under the line will be the **same**.

NO Air Bag

Large force.

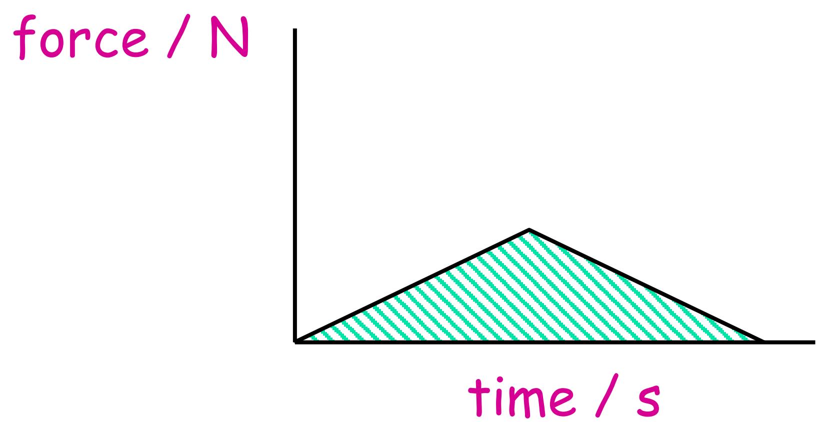
Short time.



AIR BAG

Small force.

Long time.

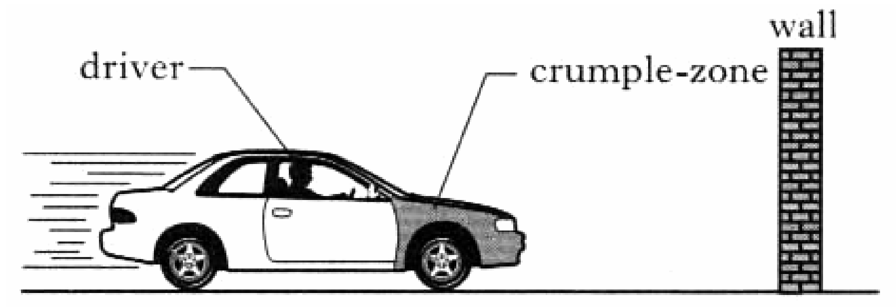


area under each graph is the same

Air bags **decrease** the **rate of change in momentum**.

Crumple Zone

A car is designed with a "crumple zone" so that the front of the car collapses during impact.



The purpose of the crumple-zone is to

- A decrease the driver's change in momentum per second
- B increase the driver's change in momentum per second
- C decrease the driver's final velocity
- D increase the driver's total change in momentum
- E decrease the driver's total change in momentum.

Less damage is caused if the change in momentum is over a long period of time.

Top Gear - Lorry Challenge

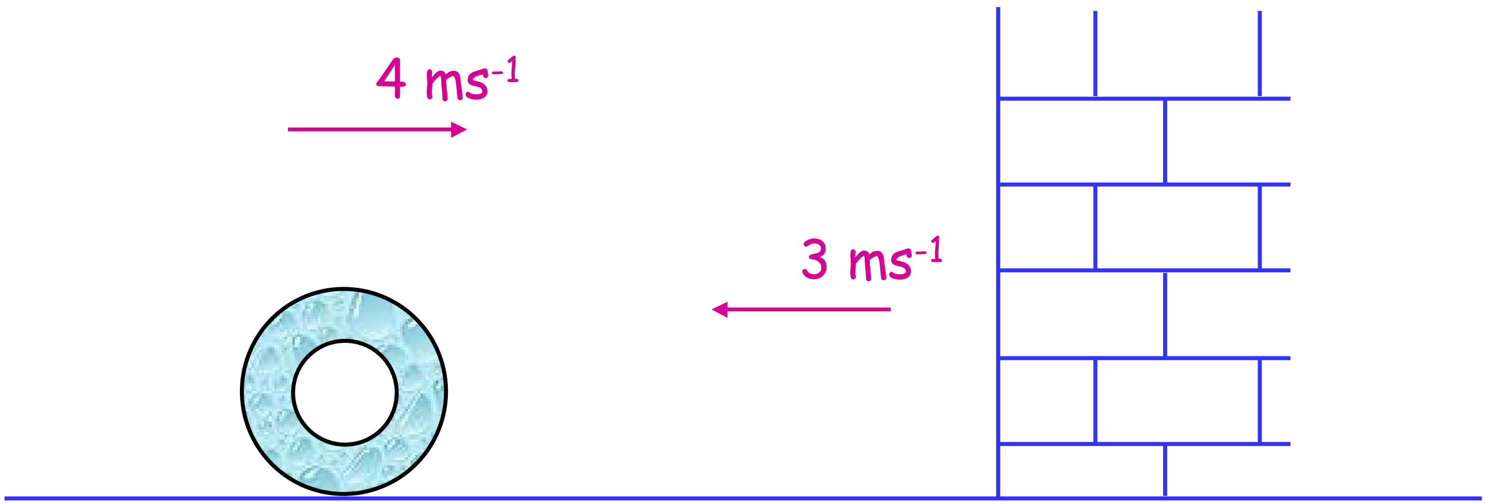
Worksheet – Impulse and Change In Momentum

Q1 - Q16

Rebounds

Example 1

A 5 kg tyre hits a wall at 4 ms^{-1} and rebounds at 3 ms^{-1}.



Calculate the change in momentum of the tyre.

$$\text{change in momentum} = mv - mu$$

$$= 5 \times (-3) - 5 \times 4$$

$$= -15 - 20$$

$$= -35 \text{ kg ms}^{-1}$$