DENSITY AND PRESSURE

Density

The mass per unit volume of a substance is called the density, r.
(The symbol, ρ, is the Greek letter rho).

$$\rho = \frac{m}{V}$$

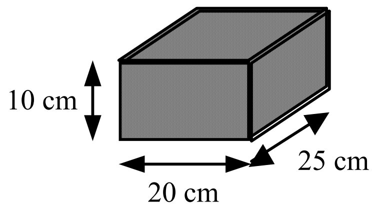
ρ = density in kilograms per cubic metre, kg m^{-3}
m = mass in kilograms, kg
V = volume in cubic metres, m^3

Example
Calculate the density of a 10 kg block of carbon measuring 10 cm by 20 cm by 25 cm.

First, calculate volume, V, in m^3 : $V = 0.1 \times 0.2 \times 0.25 = 0.005\ m^3$

ρ = ?
m = 10 kg
$V = 0.005\ m^3$

$$\rho = \frac{m}{V} = \frac{10}{0.005}$$
$$= 2000\ kg\ m^{-3}$$



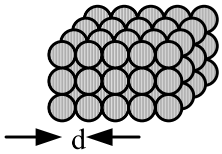
Densities of Solids, Liquids and Gases

From the table opposite, it can be seen that the relative magnitude of the densities of solids and liquids are similar but the relative magnitude of gases are smaller by a factor of 1000.
When a solid melts to a liquid, there is little relative change in volume due to expansion. The densities of liquids and solids have similar magnitudes.
When a liquid evaporates to a gas, there is a large relative change in volume due to the expansion of the material. The volume of a gas is approximately 1000 times greater than the volume of the same mass of the solid or liquid form of the substance.
The densities of gases are smaller than the densities of solids and liquids by a factor of approximately 1000.

Substance	Density (kg m^{-3})
Ice	920
Water	1000
Steam	0.9
Aluminium	2700
Iron	7860
Perspex	1190
Ethanol	791
Olive oil	915
Vinegar	1050
Oxygen	1.43
Nitrogen	1.25

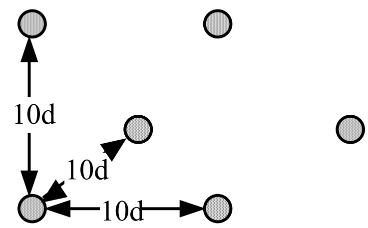
It follows, therefore, that the spacing of the particles is a gas must be approximately 10 times greater than in a liquid or solid.

Solid or liquid



Volume occupied by each particle
= d^3

Gas



Volume occupied by each particle
= $(10d)^3 = 1000\ d^3$

Pressure

Pressure on a surface is defined as the force acting normal (perpendicular) to the surface.

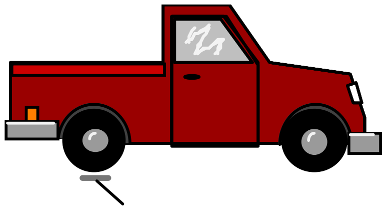
$$p = \frac{F}{A}$$

p = pressure in pascals, Pa
F = normal force in newtons, N
A = area in square metres, m^2

1 pascal is equivalent to 1 newton per square metre; ie 1 Pa = 1 N m^{-2}.

Example

Calculate the pressure exerted on the ground by a truck of mass 1600 kg if each wheel has an area of 0.02 m^2 *in contact* with the ground.



Area = 0.02 m^2

Total area A = 4 × 0.02 = 0.08 m^2

Normal force F = weight of truck = mg = 1600 × 9.8 = 15680 N

p = ?
F = 15680 N
A = 0.08 m2

$$p = \frac{F}{A} \quad \frac{15680}{0.08}$$

= 196,000 Pa or 196 kPa

Pressure In Fluids

Fluid is a general term which describes liquids and gases. Any equations that apply to liquids at rest equally apply to gases at rest.
The pressure at a point in a fluid at rest of density ρ, depth h below the surface, is given by

$$p = h \rho g$$

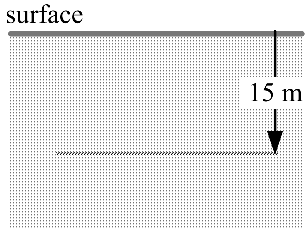
p = pressure in pascals, Pa
h = depth in metres, m
ρ = density of the fluid in kg m^{-3}
g = gravitational field strength in N kg^{-1}

Example

Calculate the pressure due to the water at a depth of 15 m in water.

p = ?
h = 15 m
ρ_{water} *= 1000 kg* m^{-3}
g = 9.8 N kg-1

p = h ρ g
= 15 × 1000 × 9.8
= 147000 Pa



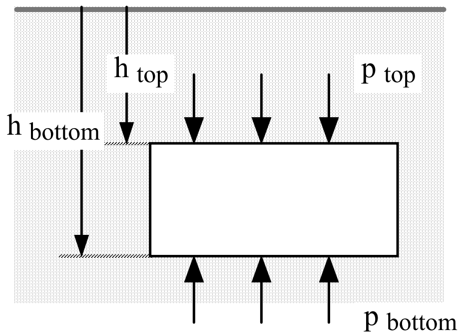
Buoyancy Force (Upthrust)

When a body is immersed in a fluid, it appears to "lose" weight. The body experiences an upwards force due to being immersed in the fluid. This upwards force is called an **upthrust.**

This upthrust or buoyancy force can be explained in terms of the forces acting on the body due to the pressure acting on each of the surfaces of the body.

Pressure on the top surface $p_{top} = h_{top}\ \rho\ g$
Pressure on bottom surface $p_{bottom} = h_{bottom}\ \rho\ g$

The bottom surface of the body is at a greater depth than the top surface, therefore the pressure on the bottom surface is greater than on the top surface. This results in a net force upwards on the body due to the liquid. This upward force is called the upthrust.



Notice that the buoyancy force (upthrust) on an object depends on the **difference** in the pressure on the top and bottom of the object. Hence the value of this buoyance force does **not** depend on the depth of the object under the surface.

GAS LAWS

Kinetic Theory of Gases

The kinetic theory tries to explain the behaviour of gases using a model. The model considers a gas to be composed of a large number of very small particles which are far apart and which move randomly at high speeds, colliding elastically with everything they meet.

Volume The volume of a gas is taken as the volume of the container. The volume occupied by the gas particles themselves is considered so small as to be negligible.

Temperature The temperature of a gas depends on the kinetic energy of the gas particles. The faster the particles move, the greater their kinetic energy and the higher the temperature.

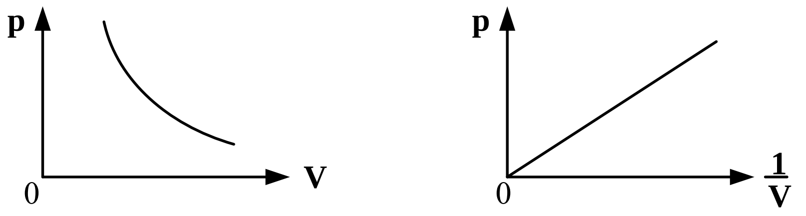
Pressure The pressure of a gas is caused by the particles colliding with the walls of the container. The more frequent these collisions or the more violent these collisions, the greater will be the pressure.

Relationship Between Pressure and Volume of a Gas

For a fixed mass of gas at a constant temperature, the pressure of a gas is inversely proportional to its volume.

$p \propto \frac{1}{V}$ $\quad$ $p \times V = \text{constant}$ $\quad$ $p_1 V_1 = p_2 V_2$

Graph

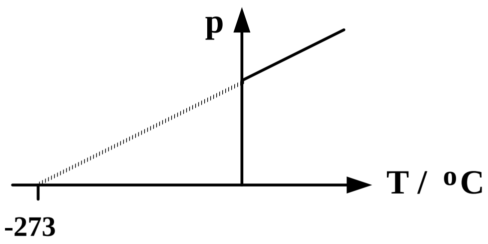


Example
The pressure of a gas enclosed in a cylinder by a piston changes from 80 kPa to 200 kPa. If there is no change in temperature and the initial volume was 25 litres, calculate the new volume.

$p_1 = 80$ kPa	$p_1 V_1 = p_2 V_2$
$V_1 = 25$ litres	$80 \times 25 = 200 \times V_2$
$p_2 = 200$ kPa	$V_2 = 10$ litres
$V_2 = ?$	

Relationship Between Pressure and Temperature of a Gas

If a graph is drawn of pressure against temperature in degrees celsius for a fixed mass of gas at a constant volume, the graph is a straight line which does not pass through the origin. When the graph is extended until the pressure reaches zero, it crosses the temperature axis at -273 °C. This is true for all gases.

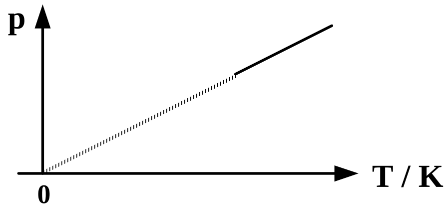


Kelvin Temperature Scale

-273°C is called **absolute zero** and is the zero on the kelvin temperature scale. At a temperature of absolute zero, 0 K, all particle motion stops and this is therefore the lowest possible temperature.
One division on the kelvin temperature scale is the same size as one division on the celsius temperature scale, i.e. temperature **differences** are the same in kelvin as in degrees celsius, e.g. a temperature increase of 10°C is the same as a temperature increase of 10 K.
Note the unit of the kelvin scale is the kelvin, K, **not** degrees kelvin, °K!

Converting Temperatures Between °C and K

Converting °C to K add 273
Converting K to °C subtract 273
If the graph of pressure against temperature is drawn using the kelvin temperature scale, zero on the graph is the zero on the kelvin temperature scale and the graph now goes through the origin.



For a fixed mass of gas at a constant volume, the pressure of a gas is directly proportional to its temperature measured in kelvin (K).

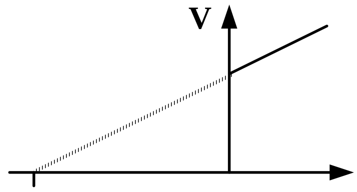
$p \propto T$ $\frac{p}{T} = \text{constant}$ $\frac{p_1}{T_1} = \frac{p_2}{T_2}$

Example
Hydrogen in a sealed container at 27 °C has a pressure of 1.8 × 105 Pa. If it is heated to a temperature of 77 °C, what will be its new pressure?

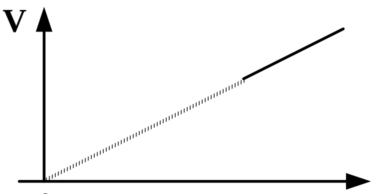
$p_1 = 1.8 \times 10^5$ *Pa*
$T_1 = 27$ *°C* $= 300$ *K*
$p_2 = ?$
$T_2 = 77$ °C $= 350$ K $p_2 = 2.1 \times 10^5$ Pa

Relationship Between Volume and Temperature of a Gas

If a graph is drawn of volume against temperature, in degrees celsius, for a fixed mass of gas at a constant pressure, the graph is a straight line which does not pass through the origin. When the graph is extended until the volume reaches zero, again it crosses the temperature axis at -273 °C. This is true for all gases.



If the graph of volume against temperature is drawn using the kelvin temperature scale, the graph now goes through the origin.



For a fixed mass of gas at a constant pressure, the volume of a gas is directly proportional to its temperature measured in kelvin (K).

$$V \infty T \qquad \frac{V}{T} = \text{constant} \qquad \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Example

400 cm^3 of air is at a temperature of 20 °C. At what temperature will the volume be 500 cm^3 if the air pressure does not change?

$V_1 = 400$ cm^3

$T_1 = 20$ °C $= 293$ K

$V_2 = 500$ cm^3

$T_2 = ?$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \qquad \frac{400}{293} = \frac{500}{T_2}$$

$T_2 = 366$ K $= 93$ °C (convert back to temperature scale in the question)

Combined Gas Equation

By combining the above three relationships, the following relationship for the pressure, volume and temperature of a fixed mass of gas is true for all gases.

$$\frac{p \times V}{1} = \text{constant} \qquad \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

Example

A balloon contains 1.5 m^3 of helium at a pressure of 100 kPa and at a temperature of 27 °C. If the pressure is increased to 250 kPa at a temperature of 127 °C, calculate the new volume of the balloon.

p_1 = 100 kPa
V_1 = 1.5 m^3
T_1 = 27 °C = 300 K
p_2 = 250 kPa
V_2 = ?
T_2 = 127 °C = 400 K

$$\frac{100 \times 1.5}{300} = \frac{200 \times V_2}{400}$$

V_2 = 0.8 m^3

Gas Laws and the Kinetic Theory of Gases

Pressure - Volume (constant mass and temperature)

Consider a volume V of gas at a pressure p. If the volume of the container is reduced without a change in temperature, the particles of the gas will hit the walls of the container more often (but not any harder as their average kinetic energy has not changed). This will produce a larger force on the container walls. The area of the container walls will also reduce with reduced volume.
As volume decreases, then the force increases and area decreases resulting in, from the definition of pressure, an increase in pressure,
i.e. volume decreases hence pressure increases and vice versa.

Pressure - Temperature (constant mass and volume)
Consider a gas at a pressure p and temperature T. If the temperature of the gas is increased, the kinetic energy and hence speed of the particles of the gas increases. The particles collide with the container walls more violently and more often. This will produce a larger force on the container walls.
As temperature increases, then the force increases resulting in, from the definition of pressure, an increase in pressure,
i.e. temperature increases hence pressure increases and vice versa.

Volume - Temperature (constant mass and pressure)
Consider a volume V of gas at a temperature T. If the temperature of the gas is increased, the kinetic energy and hence speed of the particles of the gas increases. If the volume was to remain constant, an increase in pressure would result as explained above. If the pressure is to remain constant, then the volume of the gas must increase to increase the area of the container walls that the increased force is acting on, i.e. volume decreases hence pressure increases and vice versa.